ACTICAL GEOMETRY

FOR BEGINNERS

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AND
F. W. DOBBS

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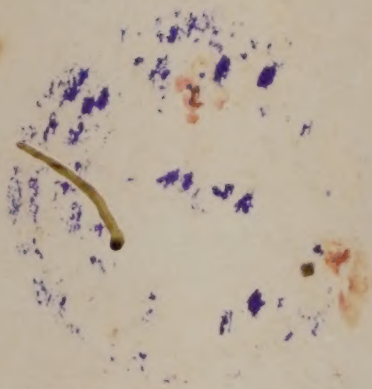

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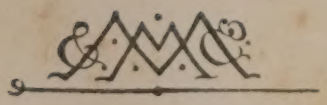

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ASSISTANT MASTERS AT ETON COLLEGE

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Answers to the Miscellaneous Examples will be supplied to *bonâ fide* teachers gratis on application to the publishers.

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PREFACE.

It is now agreed that the theory of geometry should be preceded by a course of practical work, by which the student is made familiar with the simplest figures, and is led to discover for himself some of those truths which are the groundwork of geometrical theory. Afterwards, practical drawing should be combined with formal logical proofs, partly because accurate figures are necessary to clear reasoning, and partly because careful drawing is in itself of educational value. By the time the student has finished Euclid's first book, or Part I. of his modern substitutes, he should be able to rely on deductive reasoning.

The authors have prepared what they hope will be something more than a set of metrical questions on certain propositions. They feel that such practical work should be in itself an educating force, building up a system of geometry—each example amplifying the preceding and adding to the student's knowledge. They have kept almost entirely to the subject matter of Euclid's first book; for it seemed wiser to avoid trespassing on later principles and thereby spoiling for the student, by a premature acquaintance, the joys of subsequent discoveries.

The book is divided into two Parts. Part I. is intended to precede any knowledge of theoretical geometry. It begins with measurements along straight lines and with the drawing of simple figures. From circles the student is led to construct his own protractor, to make and measure angles, and to discover the chief properties of lines and angles. The remainder of Part I. deals with the simplest constructions—the bisecting of lines and angles, and the drawing of perpendiculars. In each chapter

the full construction is gradually evolved in successive examples, and how much of the construction is essential, and how much accidental is pointed out. If it seems preferable to teach the elementary notions of area before beginning theoretical geometry, a short explanation of the use of the set square is needed before commencing Exercises XXII. and XXIII.

Part II. is intended to be used concurrently with a course of theory. Formal proofs as well as graphical verifications are required. Here the treatment is fuller than in Part I., and alternative constructions that possess intrinsic merit are included. The measurement of areas is dealt with at considerable length, as false ideas arise from an arithmetical treatment of the subject which excludes the drawing of explanatory figures. Part II. ends with the division of a line into equal parts and the construction of scales.

Two hundred miscellaneous examples, graduated and arranged in sets of six, are added. These are followed by twenty-six patterns, which it is hoped will attract the student and lead him to invent similar designs for himself. An index of the principal theorems with references to Mr. Allcock's *Theoretical Geometry for Beginners*, and cross-references to Euclid, is appended. Blank pages are placed at the end of the book, where the student can make notes of any laws he discovers for himself.

The authors wish to express their indebtedness to Mr. C. H. Allcock, at whose suggestion this work was undertaken, and gratefully to acknowledge his kindness in giving them the benefit of his experience. They would also like to place on record their appreciation of the efforts of all those, who during the last few years have been improving the methods of teaching elementary mathematics.

V. L. N. F.
F. W. D.

ETON COLLEGE.

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TABLES OF LENGTH AND AREA.

1 millimetre (mm.) $= \frac{1}{1000}$ metre (m.)
1 centimetre (cm.) $= \frac{1}{100}$ m.
1 decimetre (dm.) $= \frac{1}{10}$ m.
1 decametre (Dm.) = 10 m.
1 hectometre (Hm.) = 100 m.
1 kilometre (Km.) = 1000 m.

1 hectare = 10000 sq. metres.

In surveying the following units are used:

1 chain = 22 yards.
100 links = 1 chain.
10 chains = 1 furlong.
10 sq. chains = 1 acre.

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PART I.

EXERCISE I.

ROUGH MEASUREMENT OF STRAIGHT LINES.

Conversion of centimetres to inches, and vice versâ.

1. Measure in inches and in centimetres the length of a page of this book.

2. Measure the width of the desk at which you are working, both in inches and in centimetres. Measure also its thickness.

3. Look at your ruler and see *roughly* how many centimetres make 6 inches. From this state *roughly* your height in cm. Give your height also in metres.

4. If a cricket bat is nearly 3 feet long, what is roughly its length in cm.? Express this length also in dm. and mm.

5. Give the length of a cricket pitch in metres.

6. If cricket stumps are 27″ long, give their length *roughly* in cm.

7. Guess the length and breadth of the room you are in. Give your answer in (1) feet, (2) metres.

8. Name various objects that you think are about 1 cm. long, wide or thick.

9. Name various objects about a metre long, broad or high.

10. Name various objects about a decimetre long, broad or high.

11. Name various objects a millimetre thick.

12. The diameter of a Fives ball is $1\frac{1}{4}''$, of a racquet ball $1\frac{1}{8}''$, of a cricket ball $3''$, of an Association football $9''$. Express these lengths in centimetres.

13. What units of length (in the Metric system) would you use for measuring (*a*) a picture frame, (*b*) the distance between two towns, (*c*) a geometrical pattern, (*d*) the wall of a room, (*e*) the thickness of a wire, (*f*) the hands of a watch?

EXERCISE II.

MEASUREMENT OF STRAIGHT LINES IN INCHES AND CENTIMETRES (TO TWO PLACES OF DECIMALS).

1. Measure exactly in *inches*, with your dividers, *XA*, *XB*, *XC* ... etc.; when necessary try to estimate the hundredth part of an inch.

2. Measure in *centimetres*, with your dividers, *YM*, *YN*, *YO* ... etc.; estimate as well as you can tenths of millimetres.

3. In the figure of question 1:

   (*a*) Measure *XC*, *CB*, *BK* and *XK* in inches. Compare the values of $XC+CB+BK$ and *XK*.

   (*b*) Measure *XE*, *EB*, *BL*, *LA* and *XA* in inches. Compare the values of $XE+EB+BL+LA$ and *XA*.

   (*c*) Measure *FC*, *CL*, *LH* and *FH* in inches, and compare the values of $FC+CL+LH$ and *FH*.

4. In the figure of question 2:

   (*a*) Measure *YP*, *PQ*, *QN* and *YN* in centimetres, and compare the values of $YP+PQ+QN$ and *YN*.

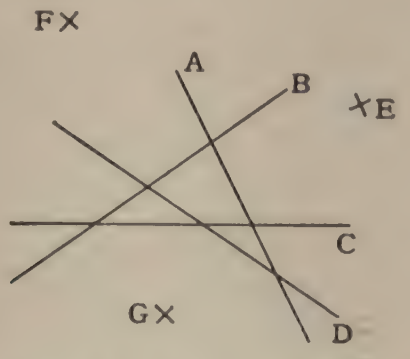
   (*b*) Measure *RS*, *SQ*, *QM* and *RM* in centimetres, and compare the values of $RS+SQ+QM$ and *RM*.

   (*c*) Measure *YR*, *RP*, *PM*, *MN* and *YN*, and compare the values of $YR+RP+PM+MN$ and *YN*.

5. Question 3 may be repeated, with measurements in cm.

6. Question 4 may be repeated, with measurements in inches.

7. Compare, with your dividers, (i) the straight lines A, B, C, D and (ii) the distances between the points E and F, F and G, G and E, and state the greatest and least in each case.



8. Draw a straight line AB 5·3″. Step a length of 2 cm. along it, until the remainder is less than a step. Measure the remainder in millimetres.

9. Draw a straight line XY 12 cm. Step a length ·7″ along it, until the remainder is less than a step. Measure the remainder in decimals of an inch.

10. (*Mental.*) If one mile is represented on a map by one inch, what distances will be shown by 6″, 3″, 4·5″, 3·6″, 2·7″? What would these lengths represent, if the scale of the map were (*a*) two inches to the mile, (*b*) six inches to the mile?

11. (*Mental.*) By what lengths would a distance of eight miles be represented in the following scales: an inch to the mile; four inches to the mile; six inches to the mile; two miles to the inch; four miles to the inch; ten miles to the inch?

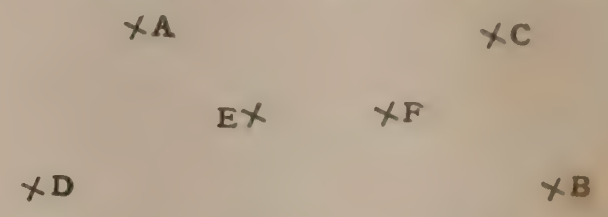
12. Using a scale in which 1 mile is represented by $\frac{1}{2}$″, draw lines to show 6 miles; 9 miles; 5 miles.

13. Draw three lines to represent 16 miles, 10 miles and 6 miles in the scale of 4 miles to the inch.

14. In the plan of a garden one yard is represented by one tenth of an inch. Draw straight lines to show the following measurements: 60 yards (the length of a lawn), 45 yards (its breadth), 26 yards (the length of a tennis court), 12 yards (its width), 2 yards (the breadth of a path).

15. The plan of a golf course is made on the scale of 100 yds. to the inch. Draw lines to represent the lengths of the following holes: (*a*) 400 yards, (*b*) 350 yards, (*c*) 380 yards, (*d*) 160 yards, (*e*) 570 yards, (*f*) 365 yards, (*g*) 205 yards.

16. Measure in inches and in centimetres the distances between the following points: (1) *A* and *B*, (2) *C* and *D*, (3) *A* and *F*, (4) *C* and *E*.



17. Open your dividers to measure exactly one inch. Measure this distance on the centimetre scale. How many centimetres make 1 inch?

18. Draw three straight lines (*A*, *B* and *C*) *exactly* 3″, 4″ and 5″ long. Measure each in cm. and make a table of your results as in the margin.

From your table calculate the number of cm. in 1 inch to 3 places of decimals.

Take the average of these results.

Line	Length in inches	Length in cm.	No. of cm. in 1 inch by calculation
A	3		
B	4		
C	5		
			3
Average			

NOTE.—*In any case your measurements can never be absolutely correct. You are liable to make small errors, but these errors are of less importance in large measurements. You should notice by comparing questions* 17 *and* 18 *that you get more accurate results by taking a long line.*

Also notice that still greater accuracy is obtained by taking the average of several results.

19. Open your dividers to measure exactly 1 cm. Measure this distance on the inch scale. What decimal of an inch = 1 cm.?

20. In the same way express 1 dm. in inches. Hence obtain 1 cm. as the decimal of an inch, and notice how much more accurate a result is obtained by measuring 10 cm. than by measuring 1 cm.

21. Draw three straight lines (*A*, *B* and *C*) 8 cm., 10 cm. and 12 cm. long. Measure each in inches, and tabulate your results as in question 18. From your table calculate the number of inches in one cm. Take the average.

EXERCISE III.

CIRCLES.

1. Draw four concentric circles with radii 5, 4, 3 and 2 cm.

N.B.—Concentric circles have the same centre.

2. Draw a straight line 3″ long, and with its extremities as centres draw two circles of radii 2″ and 1·5″ respectively.

3. Draw two equal circles with radii 1·5″ and having their centres 2″ apart.

4. Draw two circles with radii 1·4″ and 1·8″ and centres 2·2″ apart.

5. Draw a straight line 7 cm. long, and with its extremities as centres describe circles with radii 3 cm. and 4 cm.

6. Draw two circles with radii 2·6 cm. and 3·5 cm. and centres 6·1 cm. apart.

7. Notice that the circles in questions 5 and 6 touch one another *externally.*

If circles of radii 1·8″ and 1·3″ touch one another externally, what must be the distance between their centres? Draw a figure.

8. Draw two circles with radii 4 and 5 cm. touching each other externally.

9. Draw a straight line 1″ long, and with its extremities as centres describe circles with radii 1·3″ and 2·3″.

10. Draw two circles with radii 5 cm. and 2 cm. and with centres 3 cm. apart.

11. Notice that the circles in questions 9 and 10 touch one another *internally.*

What must be the distance between the centres of two circles of radii 8 cm. and 5 cm. if they touch one another internally? Draw a figure.

12. Draw two circles with radii 1·7″ and 2·5″ to touch one another internally.

13. Draw a circle of radius 2·5″. Draw two circles of radius 1″ to touch this circle, one internally, the other externally, at the same point.

14. Along a straight line (about 15 cm. long) step distances of 2 cm. with your dividers, and alternately above and below the line describe semicircles of 1 cm. radius to form a continuous curve.

15. On a straight line take two points A and B 1 cm. apart. With centre A and radii 1, 3, 5 ... etc. cm. describe semicircles above the line, and with centre B and radii 2, 4, 6 ... etc. cm. describe semicircles below the line. These will form a spiral.

16. Draw pattern No. 1. (See pages 76, 77.)

17. Draw pattern No. 2.

18. Draw pattern No. 2, but *complete* all the circles.

19. Draw pattern No. 3.

20. Two forts are situated at a distance of $4\frac{1}{2}$ miles. The one (A) has guns which carry 3 miles and the other (B) has guns which carry $3\frac{1}{2}$ miles. Draw a diagram to show the area commanded by both forts. [Represent a mile by an inch.]

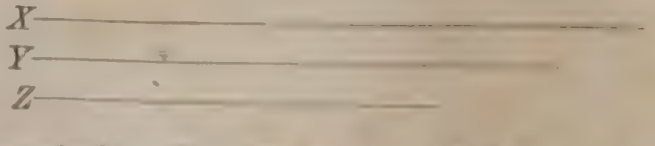
21. A donkey in a circular field (100 metres in diameter) is tethered to a stake 43 metres from the centre. The length of the rope is 12 metres. Draw a diagram showing the area of the grass he can eat. [Represent 10 metres by 1 cm.]

22. In a circular island, 300 yards in diameter, is a circular pond, 60 yards in diameter, whose centre is 75 yards from the centre of the island. Draw a diagram to show in what part of the island a man must stand in order to be able to throw stones into the pond. He can just throw 90 yards. [Scale 30 yds. to 1 cm.]

EXERCISE IV.

TRIANGLES WITH GIVEN SIDES.

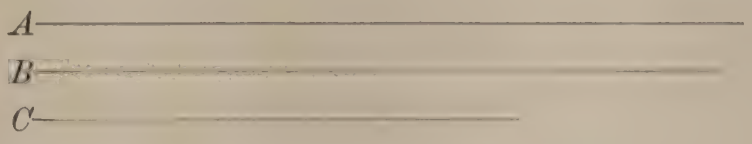
1. Draw a line AB equal in length to X. With centre A and radius equal to Y describe a circle, and with centre B and radius equal to Z describe another circle. Let C be one of the points



in which the circles cut. Join AC, BC. Compare the lengths AC and Y, BC and Z.

2. Repeat the construction of question 1, but instead of the complete circles draw small arcs, just sufficient to show their point of intersection.

3. Draw a triangle with sides equal to the three given straight lines A, B, C.



4. Draw a triangle with sides 5″, 4″ and 3″.

5. Draw an isosceles triangle with sides $3\frac{1}{2}$″, $3\frac{1}{2}$″ and 2″.

6. Draw an equilateral triangle with sides 3″. Measure its sides in centimetres.

7. Draw the triangles whose sides are given below; and in each case from the *appearance* of the triangle state whether it is acute-angled, right-angled, or obtuse-angled.

(a) 7 cm., 8 cm., 9 cm.

(b) 10 cm., 8 cm., 6 cm.

(c) 4·9″, 2·5″, 2·7″.

(d) 12·3 cm., 6·5 cm., 8·1 cm.

(e) 4·4″, 5·5″, 3·3″.

(f) 4·2″, 4·1″, 4·7″.

(g) 87 mm., 72 mm., 23 mm.

(h) 4·7″, 3·6″, 2″.

8. Draw triangles with sides

(i) 10 cms., 6 cms., 8 cms.

(ii) 10 cms., 6 cms., 4 cms.

(iii) 10 cms., 6 cms., 2 cms.

Why are the last two cases impossible? What condition must hold between the lengths of 3 lines if they are to form a triangle?

9. Draw in various positions two or more triangles with sides 3·2″, 3·7″, 4·3″. Notice that these triangles are exactly alike; this can be

seen best by cutting them out and placing them one on another. What law relates to two triangles which have the three sides of the one equal to the three sides of the other, each to each?

10. On a base 2″ long draw an isosceles triangle whose equal sides are 3″ long. On the same base draw a second isosceles triangle exactly like the first, but on the opposite side of the base.

Letter the corners of the four-sided figure so formed. Is this figure a Rhombus? Which of its angles are acute and which obtuse?

N.B.—If all the sides of a quadrilateral are equal, it is called a Rhombus.

11. On a base 7 cm. long and on opposite sides of it describe two equilateral triangles. What is the resulting quadrilateral called?

Measure the distance between the vertices of the two equilateral triangles.

12. What is the name of the quadrilateral formed by describing two isosceles triangles with equal sides 3·2 cm. on opposite sides of a base 5·2 cm.? Draw and measure the other diagonal of the quadrilateral.

13. Draw an equilateral triangle with sides 1·7″, and on each of its sides describe an equilateral triangle.

14. There are three towns *A*, *B* and *C*. The distance between *B* and *C* is 10 miles, between *C* and *A* is 7 miles, and between *A* and *B* is 6 miles. Draw a map of their positions. [Scale 2 miles to an inch.]

15. In order to map a triangular park *XYZ* the following measurements were made: *X* to *Y* 350 metres, *Y* to *Z* 290 metres, *Z* to *X* 270 metres. Draw a map of the park. [Scale 100 metres to 3 cm.]

16. Find three points at distances 2·5″, 3″, 3·5″ apart. With each of these points as centre and radius 1·5″ describe a circle.

17. Find three points *A*, *B*, *C*, such that *AB* is 9 cm., *AC* 8 cm., *BC* 7 cm. With centre *A* and radius 5 cm. describe a circle, with centre *B* and radius 4 cm. describe a second circle, and with centre *C* and radius 3 cm. describe a third circle.

18. Draw two circles radii 1·5″, 2·1″ and centres *A*, *B* 4″ apart. Find a point *C* distant 2·9″ from *A* and 3·5″ from *B*. With centre *C* describe a circle of radius 1″.

19. Draw pattern No. 4.

EXERCISE V.

BISECTION OF STRAIGHT LINES.

1. Draw a straight line AB equal to X.

X ——————————————————

Y ——————————————

With A and B as centres draw two equal circles of radius Y. Let these circles cut one another in C and D. Join CD, and let CD cut AB in E. Use your dividers to compare the lengths of AE and EB, and find whether E is the middle point of AB.

2. P ——————————————————

Q ——————————————

R ——————————

Draw a straight line AB equal to P. With its ends A and B as centres draw two circles with radii equal to Q. Join the points of intersection of the two circles by a straight line cutting AB in C. Find, by comparing AC and CB with the dividers, whether AB is bisected at C.

3. Repeat question 2, but take R instead of Q as the radius of the two circles. Why is it impossible to find the point C in this case?

4. Repeat question 2, but take Q as the radius of the circle with centre A, and R as the radius of the circle with centre B. Find the point C as before; are AC and CB equal?

N.B.—This shows that if C is to be the middle point of AB, the circles must not have different radii.

5. Draw a straight line AB 3″ long. With centre A and radius 2″ describe a circle, and with centre B and the *same* radius describe another circle. Join the points of intersection of the two circles by a straight line cutting AB in C. Measure AC and CB.

N.B.—The complete circles need not be drawn, but only sufficient arcs to find the 2 points at which the circles intersect.

6. Bisect a line 7·6 cm. long, using radii 5·4 cm.

7. Bisect a line 2·3″ long, using radii also 2·3″.

8. Bisect a line 6·2 cm. long, using radii of the same length, 6·2 cm.

9. Bisect lines of the following lengths:

(a) 11·3 cm., (b) 97 mm., (c) 3·6″, (d) 13·7 cm.

10. Divide a straight line 10·4 cm. long into four equal parts.

11. Bisect a line 4·7″ long, and bisect one of these parts again.

12. Make AB 3·7″. Take a point C distant 2·8″ from A and 1·9″ from B. Bisect AB at D. Join CD and produce it to E, making DE equal to CD (using the dividers). Join and measure AE and BE.

13. Construct a triangle with sides 4″, 4·8″, 5·6″. Bisect the sides and join the middle points of the sides.

14. Make a triangle with sides 5·4 cm., 9·5 cm., 11·5 cm. Find the middle points of the sides and join each vertex of the triangle to the middle point of the opposite side. Measure the longest of these lines.

15. Draw a triangle ABC with sides AB 4·8″, AC 3·6″ and BC 2·8″. Bisect AB at D, AC at E, BC at F and join EF, FD, DE. Measure the sides of the triangle DEF. Do you notice any relation between the sides of the triangles DEF and ABC?

16. Draw an equilateral triangle with sides 2·3″ long, and bisect the sides. On each side as diameter describe a semicircle externally.

17. Draw a triangle with sides 10 cm., 9 cm. and 8 cm. Bisect the sides, and on each side as diameter describe a semicircle internally (cutting the other sides).

18. Draw pattern No. 10.

19. Draw pattern No. 11.

EXERCISE VI.

REGULAR HEXAGON IN A CIRCLE.

1. Draw a circle with radius 2″. Step its radius round the circumference. How many steps do you take? [Compare pattern No. 1.]
Repeat the experiment with circles of various radii.

2. Draw a circle with radius 5 cm. Obtain six points at equal intervals by stepping the radius round the circumference. Join adjacent points by straight lines. This forms a regular hexagon in the circle.

3. Draw a circle with radius 1·9″. Step the radius round the circumference and describe a regular hexagon. Rule three straight lines joining each pair of opposite angles.

4. In a circle (radius 2·3″) place a regular hexagon. Join three alternate corners of the hexagon to the centre of the circle. What are the three quadrilaterals thus formed?

5. Draw a circle with radius 5·4 cm., and in it place a regular hexagon. Letter the corners *A*, *B*, *C*, *D*, *E* and *F*. Join and measure *AC*, *CE* and *EA*. What kind of triangle is *ACE*?

6. Draw pattern No. 7.

EXERCISE VII.

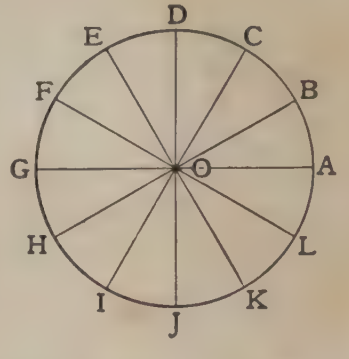
ANGLES.

Construction of a Protractor.

1. Draw a circle with any radius (about 2″). With your compasses or dividers *step* the radius round the circumference. It ought to go an exact number of times (that is, the foot of the compasses ought to return exactly to its starting point). Into how many arcs has the circumference been divided?

2. Let *A*, *C*, *E*, *G*, *I*, *K* be the six points on the circumference obtained in question 1. Join each point to the centre *O*, then the six angles so formed are all equal; this might be seen by cutting out the angles and placing them one on the top of another.

3. *By trial* with your dividers find the middle points of the six arcs, and letter them *B*, *D*, *F*, *H*, *J*, *L*.



4. By joining these points to the centre twelve angles are formed, all of which are equal.

5. As the four angles formed by the lines *AOG*, *DOJ* are equal, they must be right angles.

Therefore the angle *AOD* is a right angle and contains 90 degrees ($\angle AOD = 90°$).

How many degrees are there in each of the angles *AOB*, *BOC*, *COD*?

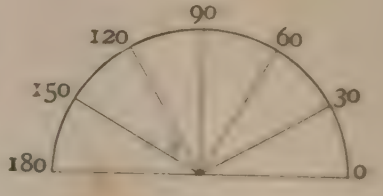
6. State the number of degrees in each of the angles *AOE*, *AOF*, *AOG*, *AOH*, etc.

7. Measure all the angles from *OA*, and write down their values as in the figure.

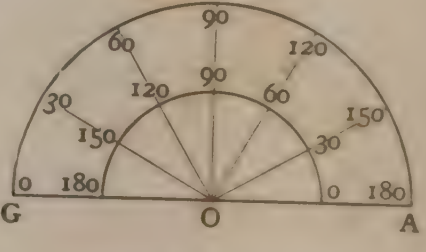
Continue in this way for all the angles in the figure till you get to the angle *AOL*.

N.B.—We are considering the $\angle AOH$ formed by a line turning round *O* from the position *OA* through each of the positions *OB*, *OC*, etc., to the position *OH*. This is not the same as the $\angle AOH$ formed by a line starting from *OA* and turning through the positions *OL*, *OK*, etc., till it arrives at *OH*. How many degrees are there in each of these angles *AOH*?

8. Draw a semicircle of radius 1″; in the same way as before find the points on its circumference and number them as in the figure.



9. With the same centre draw a larger semicircle of radius $1\frac{1}{2}$″. Mark a second set of numbers in the *reverse order*, so that we can measure angles from *OG* as well as from *OA*.



10. You have now made a **Protractor**, an instrument for measuring angles and for constructing angles.

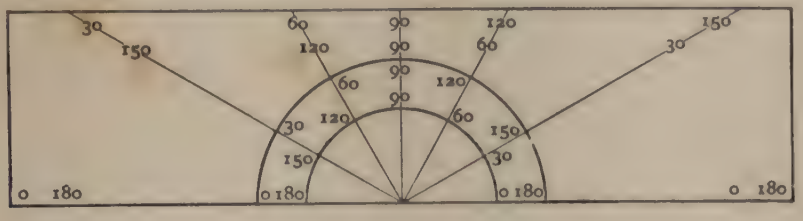
Cut your protractor out so that you can use it.

11. With your paper protractor make separate angles 30°, 60°, 90°, 120° and 150°; and write in each its size.

12. Draw several angles (pointing in various directions). Make their arms at least 3″ long. Measure each with your protractor; if you cannot do so exactly, estimate their sizes. Write in each angle its size in degrees.

13. Take a rectangular piece of paper about 8″ long and 2″ inches wide. Mark (by folding) the middle point of one of the long sides.

Place your **Semicircular Protractor** on it as in the figure. Produce the radii and fill in the numbers.



14. You have now made a **Rectangular Protractor.** Repeat questions 11 and 12 using it.

15. You should now understand how a protractor is made and how to use the one in your box of instruments.

With it measure the angles which you drew in question 12 and see how near your guesses were.

16. With your protractor make the following angles (they should point in various directions) :

20°, 80°, 90°, 30°, 150°, 120°, 40°, 35°, 55°, 75°, 175°, 125°, 95°, 93°, 98°, 72°, 121°, 39°.

In each angle write its size.

EXERCISE VIII.

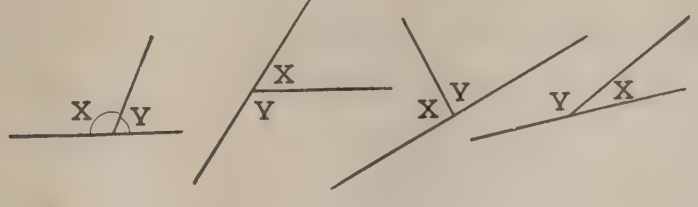
ANGLES.

The Measurement of Angles in Degrees.

N.B.—In the following questions all straight lines should be at least 2″ long, as it is easier to be accurate with large figures.

A note should be made at the end of the book of any results obtained from a number of similar experiments.

1. Draw *any* straight line meeting another straight line in various positions, as indicated below, but make your figures very much larger.



Measure the adjacent angles X and Y in each case, and find their sum.

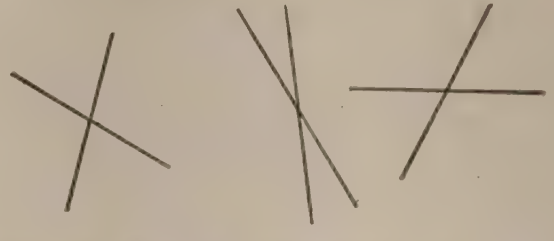
Tabulate your results as in the adjoining table, and find the average of your results.

Fig.	X	Y	X + Y
i			
ii			
iii			
iv			
		4	
Average			

N.B.—When you have found the average of your own results, it is of great advantage to compare it with those obtained by other people, and in this way to get the average of a large number of experiments.

Has any law been found which relates to the sum of the angles X and Y in every case? If so, make a note of the law at the end of the book, as suggested above.

2. Draw *any* two straight lines cutting one another in various positions, thus:



Measure all the angles, and in each write its value in degrees.

In each case (i) *tabulate* the sum of all four angles, and find the average of your results as in question 1. (ii) Compare the vertically opposite angles.

What two laws can you deduce from these results? Make a note of them.

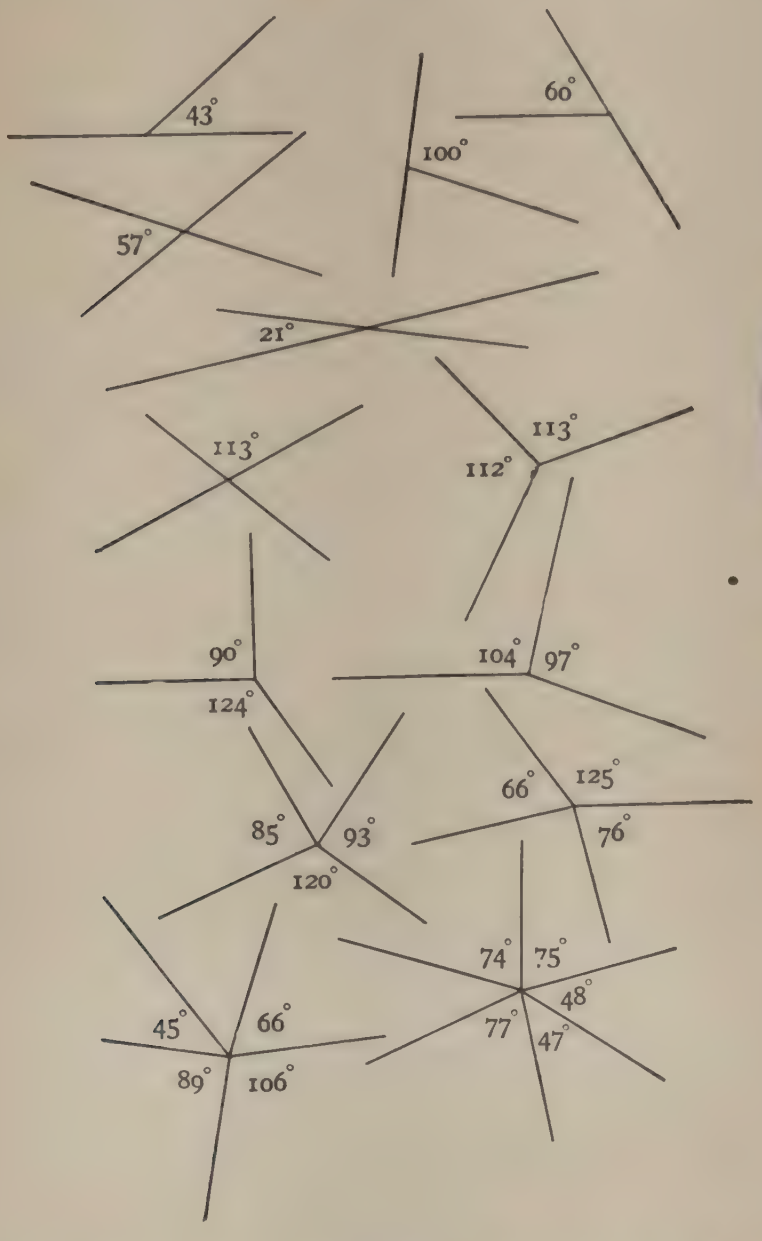
3. Draw *any* three or more straight lines meeting at a point, thus:

In each angle write its size in degrees.

In each case find the sum of all the angles: *tabulate* your results and calculate their average.

Make a note of any law you can discover from your results.

4. On a large scale draw the following figures. Measure the unknown angles; and try to check your results by using the laws you obtained in the last three questions.



5. Draw *any* (fairly large) triangle. Measure all the angles and find their sum. Repeat this experiment for triangles of various shapes (at least five times). Tabulate your results, and find the average of the sum of the angles of a triangle.

Is there any law connecting the angles of a triangle? If so, make a note of it.

6. Draw *any* equilateral triangle. Measure and compare its angles.

7. Draw *any* isosceles triangle. Measure and compare its base angles.

8. Draw *any* quadrilateral (on a large scale). Measure all the four angles. What is their sum?

Repeat these measurements for various quadrilaterals, tabulate your results and take their average.

9. Draw *any* triangle *ABC*. Produce the side *BC* to any point *D*. Measure the angles *ACD*, *ABC*, *BAC*.

Compare the size of the exterior angle *ACD* with the sum of the two interior opposite angles *ABC* and *BAC*.

This experiment may be repeated by producing the other sides of the triangle, or by taking different triangles. The results should then be tabulated and a law found connecting the exterior angle of a triangle with the sum of the two interior opposite angles.

10. Draw a semicircle, radius 2·5″. Let *AB* be the diameter. Take *any* points *C*, *D*, *E* on the circumference. Measure the angles subtended by the diameter *AB* at *C*, *D* and *E*. This experiment should be repeated, for instance, with semicircles of radii 7 cm. and 63 mm.

What is the size of an angle subtended by the diameter of a semicircle at any point on its circumference?

11. Draw a circle of radius 6·5 cm., and take any four points *A*, *B*, *C*, *D* on its circumference. Join these four points so as to form a quadrilateral *ABCD*. Measure and compare the sum of the opposite angles of the quadrilateral *ABCD*.

12. Draw three separate figures, making the two angles (i) 120° and 60°; (ii) 135° and 45°; (iii) 117° and 63° adjacent to each other.

13. Place the three angles (i) 40°, 60°, 80°; (ii) 45°, 55°, 80°; (iii) 43°, 34°, 103° adjacent to each other.

14. Place the angles 35°, 45° and 70° adjacent to each other and measure the whole angle so formed.

15. As in question 14 use a figure to add together the angles (i) 37°, 52°, 65°; (ii) 24°, 105°, 42°; (iii) 15°, 57°, 35°, 42°.

16. Draw a figure to obtain the difference between the following pairs of angles: (i) 85° and 35°; (ii) 77° and 42°; (iii) 130° and 55°; (iv) 145° and 97°; (v) 147° and 38°; (vi) 153° and 57°.

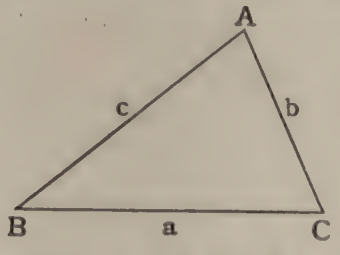
EXERCISE IX.

CONSTRUCTION OF TRIANGLES WITH GIVEN SIDES AND ANGLES.

(i) Given two sides and the contained angle.

(ii) Given two angles and the adjacent side.

N.B.—If ABC is a triangle, it is convenient and customary to name the sides a, b, c as in the figure. Thus each side and the opposite angle



are known by the same letter; a is the side opposite to the angle A, and A is the angle opposite to the side a.

In every case draw a rough figure first, and on it mark the given dimensions.

1. Construct a triangle with sides 3″ and 4″ and included angle 50°. Measure the other side and angles.

2. Draw a triangle with sides 10 cm. and 8 cm. and included angle 65°. Measure the other side and angles.

3. Draw a triangle ABC in which $a = 3{\cdot}5''$, $b = 3{\cdot}5''$, $C = 73°$. Measure A and B.

4. Draw an isosceles triangle ABC in which $a = b = 75$ mm., $C = 48°$. Produce CA to D, and CB to E; measure the angles BAD and ABE.

5. Draw a triangle ABC: given $b = 3{\cdot}2''$, $c = 2{\cdot}7''$, $A = 115°$. Measure the other angles.

6. Construct a triangle with base 4″ long and base angles 50° and 70°. Measure the other sides and angle.

7. Draw a triangle with base 12 cm. and base angles 60°. Measure the other angle and sides.

8. Construct a triangle ABC in which $a = 3{\cdot}5''$, $B = C = 67°$. Measure and compare b and c. Produce AB to D, AC to E, and measure the angles CBD and BCE.

9. Construct a triangle ABC: given $a = 9{\cdot}5$ cm., $B = 42°$, $C = 85°$. Measure A, b and c.

10. Draw a triangle ABC making $c = 3{\cdot}5''$, $B = 39°$, $A = 105°$. Measure C and b. Which is the shortest side?

11. Draw three triangles with sides 2″ and 3″, and contained angles (i) 30°, (ii) 47°, (iii) 83°. Measure the third side of each triangle.

12. Draw in different positions two triangles ABC and DEF in which $AB = DE = 10$ cm., $AC = DF = 7$ cm. and $\angle BAC = \angle EDF = 80°$. Measure and compare BC and EF, $\angle B$ and $\angle E$, $\angle C$ and $\angle F$. [The triangles could also be compared by being cut out and placed on one another.] Can you suggest any law with regard to two triangles which have two sides and the contained angles in each equal?

13. Draw in different positions two triangles KLM and PQR; given $LM = QR = 4''$, $\angle KLM = \angle PQR = 70°$, $\angle KML = \angle PRQ = 30°$. Compare KL with PQ, KM with PR, and $\angle LKM$ with $\angle QPR$. Are the triangles equal in all respects? [The triangles could also be cut out and compared.]

What law relates to triangles which have their bases and base-angles equal?

14. Draw three triangles ABC, DEF and GHK, such that $BC = 2''$, $EF = 3''$, $HK = 4''$; and $\angle B = \angle E = \angle H = 35°$, and $\angle C = \angle F = \angle K = 65°$. Measure the angles at A, D and G. Notice that these triangles are of exactly the same shape but of different sizes.

If two triangles are equiangular to one another, must they be equal in all respects?

15. Draw a straight line BC 2″ long; at its extremities make angles PBC and QCB each 72°. Let BP and CQ meet at A. Measure AB and AC. What shape is the triangle ABC?

16. Make a triangle with sides 12 cm. and 6 cm. and included angle 60°. Measure its greatest angle. What shaped triangle is it?

17. Draw a triangle ABC in which $A = 80°$, $b = c = 3{\cdot}2''$. Bisect BC at D, and join AD. Measure AD and the angles it makes with BC.

18. Draw a triangle ABC: given $a = 8{\cdot}5$ cm., $B = 73°$, $C = 50°$. Bisect the side BC in D; join and measure AD.

19. Construct a triangle ABC in which $c = 4''$, $A = 100°$, $B = 30°$. Join A to D the middle point of BC. Measure the angles BAD and CAD.

20. Draw a triangle with sides 3·2″ and 4·1″ and included angle 53°. Bisect all the sides and join the mid-points of the sides to the opposite angles.

21. On a base BC 3·8″ long draw a triangle ABC making $\angle B = 72°$ and $\angle C = 58°$. Bisect the sides AB, AC at D and E. Join and measure DE.

22. Construct a triangle with sides 6·3 cm. and 4·7 cm. and included angle 83°. On its sides, externally, describe equilateral triangles. Join the vertices of these equilateral triangles to the opposite vertices of the original triangle.

23. A ladder, whose foot is 9 ft. from the bottom of a wall, just reaches a window 40 ft. above the ground. How long is the ladder? [Scale, 10 ft. to the inch.]

24. The large hand of a clock is 2·3″ long and the small hand 1·9″. Find the distances apart of their extremities at the following times: (i) 3 o'clock, (ii) 2 o'clock, (iii) 7 o'clock.

25. An upright pole 13 metres high casts a shadow 19 metres long. How far from the top of the pole is the extremity of the shadow? Represent 2 m. by 1 cm.

26. A passenger walks into the lowest car of the Earl's Court wheel. The wheel stops when it has turned through an angle of 125°. If the radius of the wheel is 140 feet, how far is the passenger in a direct line from his starting point?

27. Two coastguards at A and B (1000 yards apart) on a straight sea-wall observe the position of a ship S by measuring the angles SAB 43° and SBA 53°. Find the distance of the ship from each man. State the scale you use.

28. A 51-foot ladder just reaches to a height of 45 feet against a wall. How far from the wall is the foot of the ladder?

EXERCISE X.

BISECTION OF ANGLES.

N.B.—It is necessary to draw fairly large figures and to use fairly large radii in your constructions; in this way greater accuracy is obtained than when small figures are drawn.

1. Draw any angle BAC. With centre A and any radius describe a circle cutting AB in P, and AC in Q. With P as centre and any

radius describe a circle, and with Q as centre and the *same* radius describe another circle. Let these two circles (with P and Q as centres) cut one another at K.

Join AK. Measure and compare the angles BAK and CAK.

2. Draw an angle BAC of 70°, making the arms AB, AC of the angle at least 3″ long. With centre A and radius 2″ describe a circle cutting AB in P, and AC in Q. With P and Q as centres and radii 1·5″ describe two circles cutting one another at K. Join AK. Measure and compare the angles BAK and CAK.

3. Repeat question 2, drawing just sufficient arcs to obtain the points P, Q and K instead of the complete circles.

4. Bisect an angle of 56°, using arcs of any radii.

5. Bisect the following angles :—(i) 30°, (ii) 140°, (iii) 84°, (iv) 112°. In each case test your accuracy by measuring the parts into which the angle has been divided.

6. Draw an angle BAC 60°. With centre A draw a circle of radius 2″ cutting AB at P, and AC at Q. Draw two circles, one with P as centre and radius 1·5″, the other with Q as centre and radius 1″; and let these circles meet in H. Join AH, and measure the angles HAB and HAC.

N.B.—This shows that in order to bisect the angle the circles with centres P and Q must have the *same* radius.

7. Bisect a right angle. (Make the right angle with your protractor.)

8. Bisect the angle BAC 85° by the straight line AD. Bisect the angle BAD by the straight line AE. What fraction is the angle BAE of the whole angle BAC?

9. Divide an angle of 128° into 4 equal parts.

10. Draw a straight line AOB. At O make the angle AOC 72°. Bisect each of the angles AOC and BOC, and measure the angle between these bisectors.

11. Draw any straight line AB meeting another straight line CD at the point B. Bisect each of the adjacent angles ABC, ABD, and measure the angle between the bisectors.

12. Draw an isosceles triangle ABC, given $A = 81°$, $b = c = 73$ mm. Bisect $\angle A$ by the line AD cutting the base BC in D. Measure and compare BD and CD.

13. Construct a triangle ABC in which $a = 10$ cm., $b = 9$ cm. and $c = 6$ cm. Draw the bisector of the angle BAC and let it cut the base BC in D. Measure BD and CD.

14. Draw a triangle ABC in which $c = 4{\cdot}3''$, $A = B = 67°$. Bisect the angles A and B and let these bisectors meet in F. Measure AF and BF.

15. Draw two straight lines AOB, COD cutting one another at O. Bisect each of the four angles so formed. What is noticeable about the bisectors of each pair of opposite angles? Measure the angle between the bisectors of each pair of adjacent angles.

16. Draw two straight lines AOB and COD making the angle AOC 70°. Bisect this angle by the straight line XOY. Notice that this line also bisects the vertically opposite angle BOD.

17. Draw a triangle with sides 10 cm., 8·3 cm. and 5·6 cm., and bisect all the angles; make these bisectors meet the opposite sides.

18. Draw any triangle and bisect all its angles; make these bisectors meet the opposite sides of the triangle.

19. Describe a circle with radius 2·3″; step the radius round the circumference, and join the six points so obtained to the centre. Bisect these six angles. [*N.B.*—This can be done by bisecting three adjacent angles.] On the circumference of the circle put the numbers I, II, III, etc., as in a clock-face.

20. With your protractor draw two lines WOE and NOS at right angles. Bisect each of the four angles between these lines. [*N.B.*—This can be done by bisecting only two angles.] Letter these bisectors to represent a Mariner's Compass.

EXERCISE XI.

PERPENDICULARS: (i) FROM INTERNAL POINTS.

1. Draw a straight line AB. In it take any point C (not near either end). With centre C and any radius describe a circle cutting AB at P and Q [only showing the necessary arcs]. With centre P and any radius greater than PC describe a circle, and with centre Q and the *same* radius describe a circle cutting the former at X. Join CX. Measure the angles ACX and BCX.

2. Draw a straight line AB about 6" long and near the middle of it take a point C. With centre C and radius 2" describe arcs cutting AB at P and Q. With P and Q as centres and radii 2" describe arcs to cut at X. Join CX. Measure the angle ACX.

3. Draw a straight line PQR making PQ 8 cm. long and QR 6 cm. long. Through Q draw a line perpendicular to PQR.

4. Draw an oblique line XYZ so that $XY = 2{\cdot}4$", and $YZ = 1{\cdot}7$". At Y draw a line at right angles to XZ.

5. In question 1, when P and Q are taken as centres, why is it necessary to use a radius greater than PQ? Endeavour to carry out the same construction using a radius less than PQ.

6. Draw a straight line AB 7 cm. long. Produce AB any distance and at B draw a line at right angles to AB.

7. Take 2 points 8 cm. apart, and through each of them draw lines perpendicular to the straight line which joins them.

8. Draw a straight line AB 3·7" long. At A and B erect perpendiculars, and from them cut off AD and BE each 2·7" long. Join and measure DE. What is the shape of the quadrilateral $ABED$?

9. Draw a circle with centre O and radius 3·2 cm. Draw any diameter AOB. Through O draw a second diameter COD at right angles to AB. Join AC, CB, BD, DA. What shape is the figure $ACBD$?

10. Draw a straight line KL 13 cm. long. At points 3 cm. from each end and on opposite sides erect perpendiculars 5 cm. long. Join the extremities of these perpendiculars. By measurement show that this line bisects KL.

11. Draw an isosceles triangle ABC in which $AB = AC = 10$ cm. and the base $BC = 6$ cm. On BA take a point D so that $BD = 4$ cm. At D draw a straight line at right angles to BA cutting AC at E. Measure CE.

12. Construct the triangle ABC in which $a = 5{\cdot}5$", $\angle B = 40°$, $\angle C = 30°$. From A draw AD at right angles to BA meeting BC at D. Measure BD.

13. Draw a straight line PQ and cut off PR 2·5" and PS 4". At R and S erect perpendiculars RT and SV. Make RT 1·5" long. Join PT and produce it to cut SV in V. Measure SV.

14. $ABCD$ is a straight line 13·2 cm. long. $AB = 5{\cdot}5$ cm. and $CD = 55$ mm. At B and C erect perpendiculars BE and CF each 7 cm. Join EF. Measure the angles at E and F. What shape is the figure $BEFC$?

15. At any point C in a straight line AB erect a perpendicular CD. On CD take any point P and with centre P and radius greater than PC describe a circle cutting AB at X and Y. Measure and compare CX and CY.

16. Make an angle BAC 47°, cut off AB 2″ and AC 1·8″. At B draw a perpendicular to AB and at C draw a perpendicular to AC. Let the perpendiculars meet at F. With centre F and radius FA describe a circle cutting AB and AC produced in P and Q respectively. Measure BP and CQ.

17. Make an angle BAC 57°. With centre A and radius 4″ describe arcs of a circle cutting BA and CA at D and E respectively. At D draw a perpendicular to AB and at E draw a perpendicular to AC. Let the perpendiculars meet at F. Measure the angle DFE.

18. Let O be the centre and AOB a diameter of a circle whose radius is 2″. Draw another diameter COD at right angles to AOB. On OC and OD cut off OE and OF each 1·5″. Join AE, EB, BF, FA. What shape is the figure $AEBF$? Measure its sides.

19. In question 18, if OE and OF are each 2·3″, what shape is $AEBF$?

20. Draw a straight line AB 6 cm. long. With centre A and radius AB describe a circle. At B draw BC at right angles to AB. Make BC 8 cm. Join CA cutting the circle at D. Measure CD.

21. Two radii, OA and OB, of a circle (radius $1\frac{1}{2}$″) are inclined at an angle 120°. Draw AD perpendicular to OA and BE perpendicular to OB. Let the perpendiculars meet at F. Measure AF, BF and the angle AFB.

22. Draw a circle with radius 2″. Draw two diameters at right angles, and bisect the angles between them. Letter the diagram to represent a Mariner's Compass.

BISECTION OF STRAIGHT LINES AT RIGHT ANGLES.

N.B.—By the construction given in Exercise V. a straight line is obtained which not only bisects, but is also perpendicular to, the given straight line.

23. Draw a straight line 4·1″ long and bisect it at right angles.

24. Take two points, A and B, 8 cm. apart and draw a straight line to bisect perpendicularly the line which joins them. On the perpen-

dicular take *any* points *P*, *Q*, *R*, and with these points successively as centres describe circles to pass through *A*. Notice that they also pass through *B*.

25. Draw a circle with radius 1·9″. Draw any straight line (not a diameter) cutting it at two points *K* and *L*. Bisect *KL* at right angles. Notice that the perpendicular bisector of a chord of a circle passes through the centre of the circle.

26. *AB* is a straight line 4″ long. Bisect it at right angles, at *O*, by the straight line *COD*. From *OC* cut off *OX* 2·1″. Join and measure *XA* and *XB*.

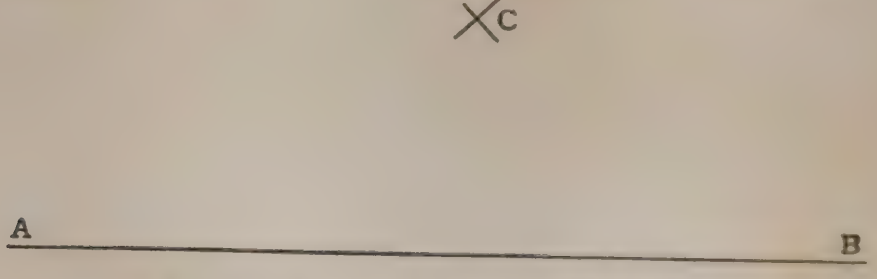
27. In the triangle *ABC*, $a = 10$ cm., $b = 8{\cdot}9$ cm., $c = 7$ cm. Bisect the sides *AB* and *AC* at right angles by straight lines meeting at *S*. Join and measure *SA*, *SB* and *SC*. With centre *S* and radius *SA* describe a circle.

28. Draw a triangle *ABC* with two sides 2·1″ and 1·7″ and included angle 135°. Bisect the two shorter sides at right angles by straight lines meeting at *S*. With centre *S* and radius *SA* describe a circle. Does it pass exactly through *B* and *C*?

EXERCISE XII.

PERPENDICULARS: (ii) FROM EXTERNAL POINTS.

1. Draw a straight line *AB* and take an external point *C* as in the figure:



With centre *C* describe a circle to cut *AB* in two points *F* and *G*. With centre *F* and radius longer than half *FG* describe a circle, and with centre *G* and the *same* radius describe another circle; let these two circles cut one another at *X* on the other side of *AB* (remote from *C*). Join *CX* cutting *AB* in *H*. Measure the angles *CHA* and *CHB*.

N.B.—Instead of complete circles, it is better to draw arcs (sufficient to obtain the points of intersection *F*, *G*, *X*).

2. As in question 1, draw a straight line AB 15 cm. long, and take a point C about 3 cm. from it.

With centre C and radius 5 cm. describe an arc cutting AB in F and G. With F and G as centres and radii 6 cm. describe two arcs cutting one another in X. Join CX and let it cut AB in H. Measure the angles CHA and CHB.

3. Make an angle BAC 30°. Along AB measure off AD 3″. From D let fall a perpendicular DE on AC. Measure DE.

4. Draw a triangle with base 12 cm. and base angles 35° and 43°. Let fall a perpendicular from the vertex on to the base.

5. Repeat question 1, but take the radii of the two circles whose centres are F and G less than half FG.

Why does the construction fail in this case?

6. Repeat question 2, but take 7 cm. as the radius of the circle with centre F, and 3 cm. as the radius of the circle with centre G. Are the angles CHA and CHB equal?

N.B.—This shows that if CH is to be drawn perpendicular to AB, the circles with centres F and G must not have different radii.

7. Draw a triangle with sides 5·3 cm., 6·5 cm. and 10 cm. Let fall a perpendicular on to the longest side from the opposite angular point.

8. Describe a circle with radius 1·8″, and draw a straight line, not a diameter, to cut the circle in any two points X and Y. Through the centre draw a line perpendicular to XY. Does this line bisect XY?

9. Draw an isosceles triangle ABC; given $a = 87$ mm., $B = C = 50°$. Through B draw BD perpendicular to AC. Measure $\angle CBD$.

10. Make an angle XAY equal to 67°. Draw AZ bisecting this angle. Along AZ measure off AB 2·7″ long. From B let fall a perpendicular BD on to AX. With centre B and radius BD describe a circle. Does this circle cut AY?

11. With radius 5 cm. describe a circle. Draw a diameter AB. Find a point P on the circumference such that AP is 6 cm. Draw PN perpendicular to AB. Measure AN.

12. From any point on the bisector of an angle let fall perpendiculars on to the arms of the angle. Compare the lengths of these perpendiculars.

13. Draw an angle PQR 55°, making QP 4·2″ and QR 3·6″. Through P and R draw lines perpendicular to QR and QP respectively. Let these perpendiculars intersect in S. Measure $\angle PSR$.

14. Draw a triangle ABC in which $a = 2{\cdot}8''$, $b = 2{\cdot}2''$, $C = 105°$. Draw the perpendicular CF on AB. Bisect BC in L. With centre L and radius LF describe a circle.

15. Draw a triangle ABC with sides $a = 3{\cdot}5''$, $b = 4{\cdot}5''$, $c = 3{\cdot}9''$. Bisect the angles A and B, and let these bisectors meet in K. From K let fall the perpendicular KN on the side BC. With K as centre and KN as radius describe a circle.

16. Draw a triangle ABC, given $a = 83$ mm., $B = 67°$, $C = 53°$. From A, B and C draw lines perpendicular to the opposite sides.

17. Find the height of a kite which is flying at the end of 300 feet of string which makes an angle of 30° with the ground.

18. The distances of a house from two consecutive milestones on a straight road are ·6 and ·7 miles. Draw a diagram to show the shortest path from the house to the road. [Scale 5″ to the mile.]

PART II.

EXERCISE XIII.

STRAIGHT LINES AND ANGLES.

1. A railway runs $2\frac{1}{2}$ miles direct from A to B and then turns to the left through an angle of 70° and goes straight on to C for $1\frac{1}{2}$ miles. Draw a diagram, 1″ to the mile, and measure the distance from A to C direct.

2. The position of a hidden treasure is fixed by the directions from a certain tree—"Go 65 feet North-East and from there measure 57 feet due East." Draw a plan showing the measurements to be made, marking the directions N, S, E and W from the tree. [Scale 1 mm. to the foot.]

3. A man walks from A 3 miles to B, there turns 45° to the right and goes on 2 miles to C, where he turns 60° to the right and proceeds to D $1\frac{1}{2}$ miles further on. How far is he now from A? [Scale 1″ to the mile.]

4. A cyclist rides from P to Q 7 miles direct, turns to the left 37°, goes on 5 miles to R, and there turns 83° again to the left and proceeds 6 miles to S. He returns from S direct to P. How far did he ride in all? [Scale 1 cm. to the mile.]

5. A ground-man marks out a football field $ABCD$ (120 yards by 80 yards) by measuring the sides AB 120 yds., BC 80 yds., CD 120 yds., and the angles B and C. His measurements are correct except for the angle B, which he makes 95° by mistake. How long does he make the side AD of the ground? [Scale 1 mm. to a yard.]

6. A lawn tennis court $PQRS$ (26 yards by 12 yds.) is marked out by measuring the sides PQ 26 yds., QR 12 yds., RS 26 yds., and by joining SP, but by mistake the angle Q is made 3° too large and the angle R 3° too small. Draw a diagram of the court as marked out. [Scale $\frac{1}{2}$ cm. to the yard.]

7. A ship sails 4 miles W, then 5 miles NE. then 3 miles W and finally $4\frac{1}{2}$ miles S. How far is she from her starting point? [Scale 1″ to the mile.]

8. Draw a diagram of the four-sided field $ABCD$ from the following data: A man walks along the boundary, from A to B 250 yds., a turn to the right 74°, from B to C 270 yds., a turn to the right 82°, from C to D 310 yds.

What is the length of the fourth side of the field? [Scale 100 yds. to the inch.]

EXERCISE XIV.

TRIANGLES AND QUADRILATERALS.

N.B.—In any triangle ABC it is customary to denote the side opposite to the angle A by a, etc. See figure, p. 17.

In every case draw a rough figure first, and on it mark the given dimensions.

Draw the triangles ABC with the following data:

1. $a = 9$ cm., $b = 5{\cdot}7$ cm., $c = 77$ mm. Measure the largest angle.
2. $a = b = 4{\cdot}3''$, $c = 2{\cdot}9''$. Measure the two largest angles.
3. $a = b = c = 3{\cdot}5''$. Measure all the angles.
4. $a = 10$ cm., $b = 94$ mm., $C = 70°$. Measure the remaining side.
5. $b = c = 3{\cdot}2''$, $A = 49°$. Measure the side a, and the angles B and C.
6. $c = 6{\cdot}2$ cm., $B = 105°$, $a = 5{\cdot}3$ cm. Measure the angles A and C.
7. $B = 45°$, $C = 55°$, $a = 4{\cdot}1''$. Measure b and c.
8. $c = 2{\cdot}2''$, $A = 43°$, $B = 111°$. Measure a.
9. $C = A = 72°$, $b = 6{\cdot}3$ cm. Measure a and c.
10. $c = 7{\cdot}3$ cm., $A = 63°$, $C = 57°$. (Begin by calculating B.)
11. $a = 2{\cdot}5''$, $A = 43°$, $B = 37°$. Measure b.
12. $b = 7{\cdot}1$ cm., $B = 37°$, $A = 73°$. Measure a.

13. Make an angle XAY 35°. From AY cut off AB 4″. With centre B and radius $2\frac{1}{2}$″ describe a circle cutting AY at C and D. Join BC and BD.

Notice that each triangle (ABC and ABD) has two sides 4″ and $2\frac{1}{2}$″ and an angle 35°. In general *two* triangles can be drawn if two sides and an angle are given, provided this angle is not the included angle.

14. Draw two triangles in which $c = 10$ cm., $a = 6{\cdot}5$ cm., $A = 30°$.

15. Draw two triangles in which $C = 41°$, $a = 4{\cdot}4$″, $c = 3$″. Measure the angles BAC. What is their sum?

16. Draw a triangle with B 50°, c 2·5″, b 3″. Why cannot *two* triangles be drawn in this case?

17. On *any* base BC draw a triangle with A 70°, B 63° and C 47°. On another base BC (of different length) draw a triangle with the same 3 angles. By taking different bases, how many triangles could you draw when only the 3 angles are given?

18. On bases 3·8″ and 3·6″ describe two triangles, whose other sides are 3″ and 3·5″. Compare the vertical angles of these triangles.

19. Draw an isosceles triangle with base 2″ and perimeter 8″. Measure the base angles.

20. Draw an isosceles triangle with base 4·2″ and vertical angle 120° (first calculate the base angles).

21. Draw a right-angled isosceles triangle on a base 4″.

22. Draw the quadrilateral $ABCD$, in which $AB = 3$″, $BC = 4$″, $CD = 3{\cdot}7$″, $DA = 3{\cdot}2$″ and the diagonal $BD = 3{\cdot}9$″. Measure the angles A and C.

23. Draw the quadrilateral $PQRS$, in which the diagonal $QS = 10$ cm., $\angle PQS = 38°$, $\angle PSQ = 75°$, $\angle RQS = 47°$ and $RQ = 7{\cdot}2$ cm. Measure PQ and $\angle PSR$.

24. Draw a rhombus with sides 2·3″ and one diagonal 1·7″. Measure the other diagonal and the angle between the diagonals. Prove that this angle should be a right angle.

25. Draw the quadrilateral $KLMN$ in which $KL = LM = 2{\cdot}7$″, $KM = 3{\cdot}4$″, $\angle NKM = \angle NMK = 64°$.

26. Draw the quadrilateral $ABCD$, in which $BA = AD = 5{\cdot}3$ cm., $BC = CD = 7{\cdot}2$ cm. and $\angle ABC = 115°$. Calculate the angle ADC and verify by measurement.

27. Draw a rhombus with sides 79 mm. and an obtuse angle 125°. Measure the longer diagonal.

28. Draw a quadrilateral whose opposite sides are equal and of lengths 4·2″ and 3·1″, and one of whose angles is 59°. Measure the other angles.

29. An endless rope has three knots at intervals of 30 ft., 40 ft. and 50 ft. Three men, each grasping a knot, pull the rope taut. Draw a diagram of the triangle formed by the rope. Measure its largest angle.

From this suggest a method for getting the angles of a lawn-tennis court "square."

30. A house A stands by the side of a straight road; 5 miles from A the road bends to the right through an angle of 45°, and at 3 miles from the bend there is another house B. Find by how much the distance from A to B is shortened by going direct, instead of by the road.

31. Draw the plan of a field whose sides are AB 300 yds., BC 280 yds., CD 240 yds., AD 260 yds., and diagonal AC 350 yds. What is the length of the other diagonal of the field? Show the position of a tree E which is distant 180 yds. from A and 140 yds. from B. How far is the tree from the other corners of the field?

EXERCISE XV.

BISECTION OF ANGLES.

1. Bisect any acute angle. Prove that your construction is correct.
2. Draw an angle of 37°. Bisect it and measure the parts.
3. Bisect an angle of 123°.
4. Divide an angle of 103° into four equal parts.
5. Make an angle equal to $\frac{3}{4}$ths of 97°.
6. Construct an angle equal to $\frac{1}{8}$th of 137°.
7. Draw a straight line AB meeting CD at B and making angles of 57° and 123° with it. Bisect the angles ABC and ABD by BX and BY respectively. Prove that the angle XBY is a right angle. Verify by measurement.
8. Two straight lines AOB and COD cut one another at a a angle. Bisect the angle AOC by OL, and the angle BOD by OK. Prove that OK

and OL are in the same straight line. Verify by laying a straight-edge along them.

N.B.—The bisector of an angle when produced also bisects the vertically opposite angle.

9. Two straight lines GOH and LOK cut one another at an angle of 43°. Bisect the angles GOK and KOH by the straight lines POR and QOS. Prove that SOR is a right angle.

10. BAC is any angle. With centre A describe an arc cutting AB at D and AC at E. With a different radius and with D and E as centres describe arcs cutting at F on the *same* side of DE as A. Prove that AF bisects the angle BAC.

Why is this method apt to be rather inaccurate in practice?

11. PQR is any angle. On its arms points D and E are taken so that $QD = QE$. On DE describe an equilateral triangle DEF (on either side of DE). Prove that QF bisects the angle PQR. Verify by measurement.

12. Draw an isosceles triangle ABC in which $a = 7{\cdot}9$ cm. and $b = c = 11{\cdot}3$ cm. Bisect the angles at B and C by straight lines meeting at O. Measure BO and CO. Prove them equal.

If BO and CO produced meet AC and AB respectively in D and E, prove that $BD = CE$. Verify by measurement.

13. Draw a triangle with sides 11 cm., 8 cm., and 12 cm. Bisect the three angles. Do the bisectors meet at a point?

14. Draw any large triangle and bisect the angles.

15. ABC is a triangle in which $A = 50°$, $b = 2{\cdot}3''$, $c = 3''$. Produce AB to D, and AC to E. Bisect the angles DBC and ECB by straight lines meeting at K. Measure the angles into which BAC is divided by AK.

16. BAD and BCD are two triangles on the same base BD, $5{\cdot}7''$, and on opposite sides of it, having BA and DC each $2{\cdot}9''$ and AD and BC each $4''$. Bisect the angle BAD by a straight line meeting BC at K and DC produced at L. Of what species are the triangles ABK, KCL, ADL (compare their sides with dividers)?

CONSTRUCTION OF ANGLES WITHOUT USING THE PROTRACTOR.

17. Draw a circle with radius $2''$. Step the radius round the circumference, obtaining the six points A, C, E, G, I, K. Join these points to the centre O. Bisect the six angles (it is only necessary to construct

three bisectors) by straight lines meeting the circumference at *B*, *D*, *F*, *H*, *J*, *L*. What is the size of each angle? What size are the angles *AOC*, *AOD*, *AOF*, *AOG*, and the reflex angles *AOH*, *AOJ*, *AOK*?

18. By bisecting the angles *AOB*, *BOC*, etc., in the previous figure construct angles of 15°, 45°, etc.

19. Using a 5 cm. radius construct the following angles at a point and with a common arm:

60°, 120°, 90°, 30°, 150°.

20. Construct at a point the following angles:

60°, 120°, 90°, 45°, 135°.

21. Construct at a point the following angles:

15°, 75°, 105°, 165°.

22. Construct the following reflex angles:

240°, 300°, 270°, 225°, 330°.

23. By construction as in question 17, find twelve points, at equal intervals, on the circumference of a circle and mark them I, II, III, etc., as in a clock-face.

24. By construction, on separate diagrams, show the positions of the hands of a watch, at the following hours: (i) 2 o'clock, (ii) 4 o'clock, (iii) 3 o'clock, (iv) 9 o'clock, (v) 4.30 o'clock, (vi) 7.30 o'clock. What is the angle between the hands in each case?

25. Make a triangle whose base is 2·5″ and base angles are 60° and 45°.

26. Draw a circle with radius 2″. By construction draw two diameters at right angles, and two others cutting them at angles of 45°. Join the extremities of the diameters so as to form a regular octagon. Measure its sides.

EXERCISE XVI.

BISECTION OF LINES.

1. Bisect a straight line 3·7″ long. Prove that your construction is correct.

2. Divide a straight line 13 cm. long into four equal parts by repeated bisection.

3. Draw a triangle ABC with base BC 3″ and base angles 57° and 65°. Bisect AB and AC at D and E respectively. Join DE. Measure and compare DE and BC.

4. On a base BC 3″ long draw *any* triangle ABC, and repeat the construction of question 3. Compare DE and BC.

5. Draw a straight line AB 7·9 cm. long. Bisect it and produce it to C, making AC half as long again as AB.

6. Make a line half as long again as 3·5″. Measure it.

7. AB is a straight line 3·1″ long. Produce it to C, making AC a quarter as long again as AB. Measure AC in centimetres.

8. Divide a straight line 4·9″ long into eight equal parts by repeated bisections.

9. Draw a straight line AB 3·8″ long. Bisect it at C by the straight line DCE cutting it at right angles. Make CD and CE each 1·9″. Join AD, DB, BE, EA. What is the figure $ADBE$? Prove that its sides are equal.

10. Draw a square with diagonal 3″.

11. Bisect each side of the square described in the previous question and *internally* describe semicircles on the sides.

12. Draw a straight line PQ 8·8 cm. long. Bisect it at right angles at S by the straight line RST. Make RS and ST each 3·3 cm. Join and measure PR, RQ, QT and TP. What is the figure $PRQT$?

13. Construct a rhombus with diagonals 4·1″ and 3·2″. Measure the obtuse angles.

14. AB is 3·4″. Bisect it at D at right angles by CDE. Make CD 1·1″ and DE 2·8″. Prove $AC = BC$ and $AE = BE$. The figure $ACBE$ is called a kite.

15. Take a straight line AB 10 cm. long. With centres A and B and radii 7 cm. describe arcs to cut *above* the line at C, and with centres A and B and radii 6 cm. describe arcs to cut *below* the line at D. Join CD, measure and compare the parts into which it divides AB. Also prove that this is a correct method for bisecting a given straight line.

16. Take a straight line AB 10 cm. long. With centres A and B and radii 7 cm. describe arcs to cut at C *above* the line and with the same centres and radii 6 cm. describe arcs to cut at D also *above* the line. Join CD and produce it to meet AB at E. Measure AE and EB. Prove

AE and EB equal. Though this is theoretically a correct method for bisecting a line, why is it rather inaccurate in practice?

17. Take a straight line AB 4″ long. On it describe a triangle ABC with AC 3″ and BC 2″; on the *opposite* side of AB describe a triangle ABD with AD 2″ and DB 3″. Join CD cutting AB at O. Measure and compare AO and OB.

18. Draw a triangle ABC with a 8·5 cm., $\angle B$ 63°, $\angle C$ 55°. Bisect the side AB and on it, as diameter, describe a circle cutting BC at D. Join AD. Measure the angle ADB.

19. Draw a triangle ABC with a 3·5″, b 2·8″ and C 57°. Bisect CB and CA at D and E; on CB as diameter draw a semicircle cutting AB at M. With centre E and radius EM describe a circle; where does this circle cut AC?

20. Draw a triangle ABC; given $a = 8$ cm., $B = 52°$, $C = 74°$. On AB as diameter describe a semicircle cutting BC at K, and AC at L. Are AK and BL at right angles to BC and CA respectively?

21. Draw *any* large quadrilateral $ABCD$. Bisect the sides at the points E, F, G and H. Measure the sides and angles of the figure $EFGH$. What kind of quadrilateral is it?

22. Draw AB 7 cm. long. Bisect it at right angles by a straight line CD. On CD take *any* point P. Measure and compare PA and PB. Prove $PA = PB$.

23. Make AB 3·8″ long. Find a point P such that $AP = BP = 2·7$″. Prove that P must lie on the line which bisects AB at right angles, and verify this by drawing the line.

24. All points equidistant from two given points A and B 5·9 cm. apart lie on a straight line. Draw this line.

N.B.—This line is called the *locus* of all points equidistant from A and B.

25. Two houses A and B are 3 miles apart and a road runs so that every point on it is equidistant from the houses. Draw a diagram of the road and the positions of the houses. Scale 1″ to the mile.

26. Draw a triangle ABC, with a 3·7″, b 2·9″ and c 3·3″. Bisect AB and AC at right angles by straight lines meeting at S. Join SA, SB and SC. Prove them equal. Verify by describing a circle with centre S and radius SA.

N.B.— This circle is called the "circum-circle" of the triangle.

27. Draw a triangle with sides $3\frac{1}{2}''$, $4''$, $4\frac{1}{2}''$ and bisect all three sides at right angles.

28. Three houses A, B and C are situated at the angular points of a triangle whose sides are 3·5 miles, 2·2 miles and 2 miles. There are 3 roads which run so that every point on the first is equidistant from B and C, every point on the second is equidistant from C and A and every point on the third is equidistant from A and B. Draw a diagram of the positions of the roads and houses. Show that the 3 roads meet at a point. How far is this point from each house?

29. Draw a triangle ABC, in which $a = 8$ cm., $b = 7{\cdot}2$ cm. and $C = 49°$. Bisect two sides at right angles by straight lines meeting at S. With centre S and radius SA describe a circle. Does it pass exactly through B and C?

30. A man has a triangular field with sides 370 yards, 300 yards and 260 yards. In it he wishes to plant a tree which shall be at equal distances from the corners of the field. Draw a diagram to show where he should plant it. How far is it from the corners? [Scale 100 yds. to the inch.]

31. Draw a triangle ABC with BC $4''$, AB $3{\cdot}7''$ and AC $2{\cdot}9''$. Find D the middle point of BC. Join and measure AD.

N.B.—The straight line joining a vertex of a triangle to the middle point of the opposite side is called a "median."

32. Draw *any* large triangle ABC. Bisect AB at F and AC at E. Draw the medians BE and CF, cutting at G. Join AG and produce it to meet BC in D. Compare BD and CD (with your dividers).

N.B.—The point of intersection, G, of the three medians is the "centre of gravity" of the triangle.

33. Draw a triangle ABC with a 9·2 cm., $\angle B$ 53°, $\angle C$ 72°. Draw the three medians, and measure the shortest.

34. Draw an arc of a circle (the extremities A, B should be at least $3''$ apart). Bisect the chord AB of the arc at right angles. Notice that this bisector also divides the arcs into two equal parts.

35. Draw a circle with radius $2{\cdot}1''$. On it with your dividers take any two points A and B $3''$ apart. Bisect the chord AB at right angles. Notice that the bisector passes through the centre of the circle.

36. Inscribe a regular hexagon in a circle, diameter 10 cm. Bisect the sides. [*N.B.*—It is only necessary to bisect 3 adjacent sides.] Join the mid-points of the sides to form another hexagon. Is it also a regular figure?

EXERCISE XVII.

PERPENDICULARS (INTERNAL).

1. Make a straight line AB 5″ long, and at a point C 2″ from one end erect a perpendicular to it. Prove that your construction is correct.

2. $ABCD$ is a straight line 13 cm. long. B is 5 cm. from A and C is 55 mm. from D. Through B and C draw perpendiculars to AD.

3. Draw a line AB 4″ long. Cut off AC 1″ and AD 2½″. At C and D erect perpendiculars CX and DY. Along CX measure off CE 1·2″ long. Produce AE to meet DY in F; and let BE cut DY in G. Measure DF and DG.

4. Two upright poles, 12 ft. and 9 ft. 6 in. high, are erected on level ground and their feet are 6 feet apart. How far apart are their tops?

5. A captive balloon, with 200 yards of rope, starts from a straight level road and rises vertically. Draw a diagram to show its course and its position when it has risen to its full height. A spectator stands 150 yards from its starting point; how far is he now from the balloon?

6. A pier 100 metres long juts out into the sea due South from the shore, which runs East and West. How far, in a direct line, from the pierhead is a coastguard who is standing on the shore 75 metres from the pier gates?

7. AB is a straight line 5″ long, and C a point in it is 2·2″ from A. Take *any* point D outside the line and 1½″ from C. With centre D and radius 1½″ describe a circle cutting AB at C and K. Join KD and produce it to meet the circle at P. Join PC. Measure the angle PCB. Note this method of constructing a perpendicular.

8. Draw two straight lines AOB and XOY inclined at an angle of 32°. Through O draw lines OC and OZ perpendicular to AB and XY respectively. Measure the angle COZ.

9. Two points A and B are 5″ apart, find C such that $AC = 4$″ and $BC = 3$″. From AC cut off AD 1·6″ long, and draw DE at right angles to AC to meet AB in E. Measure AE.

10. O is the centre and AB a diameter of the circle ACB of radius 1·8″, and $\angle COB$ is 63°. Draw CD perpendicular to OC to meet AB produced at D. Measure OD.

11. Draw a straight line AB 13 cm. long, and in it take C so that $AC = 2$ cm. With centre C and radius 5 cm. describe a circle cutting AB at D. Step the radius twice along it from D and obtain the points E and F. With E and F as centres describe arcs of equal circles to cut at G. Join CG. Measure the angle GCB.

Notice that the above is the construction for bisecting the angle between the two lines CE and CF which make 60° and 120° with CB.

N.B.—This is a convenient method for constructing a perpendicular to a given straight line from an internal point, if that point is near, or at, either extremity of the line.

12. Draw a straight line 4·3″ long. At its ends erect perpendiculars 3·2″ long. Join the extremities of the perpendiculars. What kind of quadrilateral is drawn?

13. Construct a rectangle $ABCD$, having AB 5·6 cm. and AD 11·2 cm. Measure the angles ADC and BCD. Let AC and BD intersect at O; describe a circle to pass through A, B, C, D.

14. Make a rectangle whose sides are 2·3″ and 2·9″; draw its diagonals, measure their lengths, and prove that they must be equal.

15. Construct a square with sides 3″. Measure its diagonals.

16. Construct a square with sides 8 cm. Draw its diagonals. Describe a circle to pass through its angular points.

17. (i) Bisect an angle ACB of 170°.

(ii) Bisect an angle ACB of 180°.

(iii) At any point C in the straight line AB draw a perpendicular to it.

Compare the figures. Notice that erecting a perpendicular from an internal point is the same as bisecting an angle of two right angles at that point.

18. Draw a line AB 7·5 cm. At B erect a perpendicular BZ. With centre A and radius 8·5 cm. describe an arc cutting BZ at C. Measure BC.

19. Draw a right-angled triangle with hypotenuse 2·9″ and a side 2·1″. Measure the third side.

20. Draw a right-angled triangle with a side 2·3″ and hypotenuse 4·6″. Measure its smallest angle.

21. A twelve-metre ladder leans against a vertical wall with its foot $2\frac{1}{2}$ metres from the bottom of the wall. To what height does it reach?

22. At any point A in a straight line XY erect a perpendicular AB, 7 cm. long. Draw BD and BE meeting XY in D and E and making angles of 30° with BA. What is the shape of the triangle DBE?

23. Construct an equilateral triangle of altitude 2·6″.

24. Construct an isosceles triangle of height 3·6″ and equal sides 4″. Measure the base angles.

25. On a base 6 cm. long describe an isosceles triangle of height 8·2 cm. Measure the sides, and prove them equal.

26. Draw an isosceles triangle of vertical angle 100° and altitude 55 mm. Measure the base.

27. Construct a triangle having its vertex 2″ from its base and the sides containing the vertical angle 3″ and 4″ respectively.

28. Construct a triangle of which 7 cm. is the height, and 10 cm. and 8 cm. the lengths of the sides meeting at the vertex.

29. A donkey is tethered by a rope 8 m. in length to a stake which is 5 m. from a long straight hedge. Draw a diagram showing how much of the hedge the donkey can nibble. [Scale 1 m. to 1 cm.]

30. Draw a diagram of two bicycle wheels (diameters 28″ and 20″) having their points of contact with a level road 33″ apart. How far apart are the tops of the wheels? [Lines to be drawn $\frac{1}{16}$th of the actual size.]

N.B.—The **set square** may be used for drawing perpendiculars.

This method is best when several perpendiculars have to be drawn to the same line.

Any of the previous questions (omitting 7, 11, 17) may be used as examples of this method.

EXERCISE XVIII.

PERPENDICULARS (EXTERNAL).

1. Draw a perpendicular to a given straight line from a given point outside it. Prove that your construction is correct.

2. Draw a perpendicular CH on to the straight line AB from an external point C. In HB take any 3 points P, Q, R. Write CP, CQ, CR and CH in order of magnitude.

3. From an external point C let fall a perpendicular CD on to a straight line AB. In AB take any three points H, K and L, such that DK equals DH, and DL is greater than DH. Prove that $CH = CK$, and that CD is the shortest and CL the longest of the lines CD, CK and CL.

4. Draw a triangle with sides 3″, 4″ and 5″; and draw a perpendicular to the longest side from the opposite angular point. Measure this perpendicular.

5. ABC is a triangle in which $c = 4{\cdot}3''$, $A = 35°$ and $b = 2{\cdot}7''$. From C draw CD perpendicular to AB. Measure and write down the lengths of CD and AD.

6. A kite is flying at the end of a string 360 feet in length which makes an angle of 30° with the ground. Find the height of the kite.

7. Draw an angle XAY 45°, and draw AP 4″ long and making an angle of 20° with AX. From P draw lines at right angles to AX and AY, and measure the angle between them.

8. Two men at the ends A and B of a straight sea wall 550 yards long find the position of a ship S by simultaneously observing the angles SAB and SBA to be 46° and 58°. Find the distance of the ship from the sea wall.

9. To find the height of a tower CD, a man at A observes the angle CAD to be 17°; he then walks to B, 65 metres nearer the foot of the tower, and finds the angle CBD to be 33°. Draw a diagram to find the height of the tower.

10. Let AB be any straight line and C any point outside it. In AB take *any* point D. Join CD and bisect it at E. With centre E and radius CE describe a semicircle to cut AB at F. Join CF. Measure the angle CFA.

Join EF and prove that $\angle CFD = \angle FCD + \angle FDC =$ a right angle.

N.B.—This method of drawing a perpendicular is useful when the perpendicular will fall near the end of the line.

11. ABC is *any* triangle, for convenience make a 12 cm., b 11 cm., and c 10 cm. On AB as diameter describe a semicircle cutting AC at E and BC at D. Join AD and BE cutting at O. Measure the angles at D and E.

12. ABC is a triangle with sides 4·8″, 4·3″ and 4″. Repeat the construction of the previous question. Join CO and produce it to meet AB at F. Measure the angles at F.

13. Draw a triangle with two sides 10 cm. and 11 cm. and contained angle 50°. From each angular point draw a perpendicular on to the opposite side.

N.B.—The three perpendiculars meet in a point.
This point is called the "Orthocentre" of the triangle.

14. Draw a triangle with sides 11·1 cm. and 9·2 cm. and included angle 55°. Draw a perpendicular from each angular point to the opposite side.

15. Draw *any* obtuse-angled triangle. Produce the shorter sides. From each angular point draw a perpendicular to the opposite side. When the perpendiculars are produced do they pass through a point?

16. Draw a triangle with sides 2″ and 2·7″ and included angle 115°. From each angular point draw a line at right angles to the opposite side.

17. Make an angle of 43°. Bisect it, and on the bisector take a point 2·7″ from the angular point. Let fall perpendiculars from it on to the arms of the angle. Measure and compare these perpendiculars, and prove them equal.

18. Draw a triangle *ABC* with sides 10 cm., 11 cm. and 13 cm. Bisect the angles at *A* and *B* and let these bisectors intersect at *K*. From *K* let fall perpendiculars on to the sides of the triangle *ABC*. Measure these perpendiculars, and prove them equal.

19. Draw a triangle *ABC* with two sides 4″ and 3″ and included angle 100°. Let the bisectors of any two angles meet at *K* and draw a perpendicular *KN* to one of the sides of the triangle. With *K* as centre and *KN* as radius describe a circle.

N.B.—This circle should touch the sides, and is called the "inscribed circle" of the triangle.

20. Draw a square *ABCD* with sides 3·6″. Let *AC* and *BD* cut at *K*. Draw *KN* perpendicular to *AB*. With *K* as centre and radius *KN* describe a circle.

N.B.—The **set square** may be used for drawing perpendiculars.
Any of the previous questions (omitting 10, 11, 12) may be used as examples of this method.

EXERCISE XIX.

THEOREMS RELATING TO ANGLES.

1. OA, OB, OC are three radii of a circle (radius 2″) such that the angles between each pair are equal. Calculate them, and draw the figure. Prove that the chords AB, BC, CA of the circle are equal. Check by measurement.

2. OA, OB, OC, OD, OE are five radii of a circle (radius 2″) such that the angles between each pair are equal. Draw the figure. Join AB, BC, ... etc. What is the figure $ABCDE$?

3. In the previous question, instead of joining AB, etc., draw the following lines, AC, CE, EB, BD, DA (the radii should be as faint as possible).

4. Make 8 equal angles at the centre of a circle, radius 5 cm., and join the extremities of the radii to form a regular octagon.

5. Draw a diagram of a wheel with 9 spokes.

6. ABC is a triangle in which $BC = 9$ cm., $\angle ABC = 50°$, $\angle ACB = 51°$. Which is the greater, AB or AC? Verify by measurement.

7. In the triangle ABC, $AB = 5''$, $CB = 3''$, $AC = 3{\cdot}1''$. Which of the angles CAB and CBA is the greater? Verify by measurement.

8. Within the triangle ABC any point D is taken. Join BD and CD. Which is the greater of the angles BAC, BDC? Check by measurement.

The following examples illustrate angular properties of rectilineal figures.

It is as well to draw large figures.

9. The side BC of a triangle ABC is produced to D. Measure the angles A and B. Calculate the exterior angle ACD. Check by measurement.

10. What is the sum of the angles of a triangle? Draw a figure, and check by measurement.

11. Calculate the sum of the interior angles of a quadrilateral. Check by measurement.

12. Calculate the sum of the interior angles of (i) a pentagon, (ii) a hexagon. Check by measurement.

13. In the triangle ABC the side BC is produced to D, CA to E and AB to F. What is the sum of the angles ACD, BAE, CBF? Check by measurement.

14. The sides of a quadrilateral are produced in the same sense. What is the sum of the exterior angles so formed? Check by measurement.

15. Calculate the sum of the exterior angles of (i) a pentagon, (ii) a hexagon, when their sides are produced in the same sense. Check by measurement.

16. AOB is the diameter and O the centre of a semicircle whose radius is 7 cm. On the circumference *any* point Q is taken, and AQ and QB are joined. Measure the angle AQB.

Take Q in various positions, and observe that the angle AQB is invariably a right angle. This can be proved by joining OQ, considering the isosceles triangles OAQ, OBQ, and remembering that the three angles of a triangle make up 180°.

N.B.—This Theorem, "*The angle in a semicircle is a right angle*," will be used in subsequent examples.

EXERCISE XX.

COPYING ANGLES.

1. Draw any acute angle BAC. At the point P in a straight line PQ construct an angle equal to BAC. Give the proof that your construction is correct.

2. Draw an angle of 72° with your protractor. Copy it by construction.

3. On a base $3\frac{1}{2}''$ draw a triangle with its other sides $2\frac{1}{4}''$ and $3''$. On a base of 10·5 cm. describe a triangle with angles at the base equal to the angles at the base of the original triangle.

4. Draw a triangle whose sides are $3''$, $4''$, $5''$. Construct a similar triangle with longest side 9 cm. Measure the shortest side of this triangle.

5. Draw a triangle ABC having c $5{\cdot}2''$; b $4{\cdot}7''$ and a $4''$. Construct a triangle similar to this with its longest side 9·1 cm. Measure the shortest side of this triangle.

6. Make an isosceles triangle with base 7·2 cm. and equal sides 8·7 cm., and on a base 3·6″ construct a triangle equiangular to it. Measure the sides.

7. Draw a triangle ABC, in which $c = 3{\cdot}7''$, $b = 4{\cdot}2''$, $a = 5''$. Construct a similar triangle, whose greatest side shall be 1 dm. Measure the shortest side in millimetres.

8. Make an angle of 33°, and by *construction* make an angle twice as large. Measure it.

9. Make an angle of 18°, and by construction make an angle 3 times as large. Check by measurement.

10. Draw an angle of 53°, bisect it, and construct an angle half as large again.

11. Draw an angle of 37°, and construct an angle half as large again.

12. Draw any two angles, and at a point in a straight line and on *opposite* sides of it construct angles equal to them to make an angle equal to their sum. Check your figure by measuring the 3 angles.

How would you make an angle equal to the difference of two given angles?

13. ABC is a triangle with sides a 13·7 cm., b 7·5 cm., c 9·7 cm. Make angles equal to

(i) $A - B$, (ii) $A - C$, (iii) $C - B$. Measure each.

14. Draw a triangle with a 5·3″, b 3·1″, c 3·9″. At A in AB make the angle KAB equal to B (and remote from C), and at A in AC make the angle LAC equal to C (and remote from B). Test whether KA and AL are in the same straight line by applying a straight-edge to them.

15. BAC is any acute angle and AB is 4·7″ long. At B in AB make an angle DBA equal to BAC and on the *opposite* side of AB. Will AC and DB, if produced, meet?

16. Repeat question 15, but take BAC an obtuse angle.

EXERCISE XXI.

PARALLEL STRAIGHT LINES.

1. Let AB be any straight line and C any point outside it. In AB take any point D and join DC. Make (by construction) the angle ECD alternate and equal to the angle CDB. Will EC and AB, if produced, ever meet?

2. If PQ is a given straight line and X a point outside it, take any point K in PQ, join XK and produce it to L. At X make the angle KXY equal to the angle PKL (by construction), and on the same side of XL. Prove that YX and PQ are parallel.

3. Draw a triangle ABC with A 110°, b 4 cm. and c 6 cm. Through A draw PAQ parallel to BC, by making the angle BAP equal and alternate to the angle ABC.

4. At the extremities of the straight line AB, 3·1″ long, and on the same side of it, make the angles KAB 73° and LBA 107°. Are LB and KA parallel?

5. Draw an isosceles triangle ABC with $b = c = 8$ cm. and a 5 cm. Produce BA to K. Through A draw AL parallel to BC. Prove that AL bisects the angle CAK.

6. Draw two parallel lines at a distance of 45 mm. apart.

7. Draw a straight line parallel to a given line AB, and at a distance of 2·3″ from it.

8. Draw an angle ACD 57°. Make AC 2·7″. Through A draw AB parallel to CD, using the set square. Bisect the angles BAC and DCA by straight lines meeting at K. Measure the angle at K.

9. With the set square draw two parallel straight lines about 1½″ apart. On them take any points A and B. Bisect AB at O. Through O draw any straight lines terminated by the parallels. Prove that all are bisected at O.

10. Draw an angle BAC of 63°. Bisect it by AX and on the bisector cut off AD 3·1″. Through D draw DY parallel to BA, cutting AC at Y. Prove that the triangle AYD is isosceles.

11. Draw a triangle with sides 1·7″, 2·1″ and 1·4″, and through each angular point draw a straight line parallel to the opposite side.

12. Draw a straight line AB, 2½″ long; on the same side of it make $\angle DAB$ 60° and $\angle CBA$ 117°. Along BC measure off BE 1·6″ and BF 4·5″; draw EG and FH perpendicular to BC, and meeting AD at G and H. Compare EG and FH. Are the lines AD and BC parallel?

13. Draw 2 parallel straight lines AB and CD and draw *any* straight line $HKLM$ cutting them at K and L respectively. Measure all the angles at K and L and in each write its size. Compare (i) the alternate angles, (ii) an exterior angle with the interior opposite angle on the same side of the line, and (iii) find the sum of two interior angles on the same side of the line.

14. Construct the following figures, using the set square to draw the opposite parallel sides; compare their sides and angles.

(i) A parallelogram with sides 4″ and 3″, and one angle 80°.

(ii) A parallelogram with sides 9·2 cm. and 7·3 cm., and one angle of 105°.

(iii) A rhombus with sides 3·9″ and one angle 78°.

(iv) A rhombus with sides 8·2 cm. and one angle 135°.

(v) A rectangle with sides 3·7″ and 2·3″.

(vi) A rectangle with sides 5·3 cm. and 78 mm.

(vii) A square with sides 3·1″.

15. *AB* and *CD* are two parallel straight lines each 3·4″ long. Join their extremities (towards the same parts) and prove the joins also equal and parallel. Check by measurement.

16. *PQ* and *XY* are two parallel straight lines, each 7·2 cm. long. Join their extremities to cut at *K*. Prove that the joins bisect each other.

17. Draw two parallel straight lines 4·3″ and 3·2″ long. Join their extremities towards the same parts. What kind of quadrilateral is formed?

18. *XY* is a given straight line and *A* a given point on it. Find a point *B* which shall be 1·7″ from *XY* and 2·5″ from *A*.

19. Along a level straight road, upright pillars 33 ft. and 37 ft. high, are erected at a distance 15 feet apart. Where must a third pillar 41 ft. high be erected so that the three tops may be in a straight line?

20. Construct a triangle *ABC* with base *BC* 8 cm., side *AB* 6 cm. and height 4·5 cm.

21. Draw an angle *XAY* of 67°. Find a point 1″ from *AX* and 1·3″ from *AY*.

22. Draw an angle *BAC* 73°. Find a point which is $1\frac{1}{2}$″ from each arm of the angle.

23. *ABC* is a triangle with *a* 11·5 cm., *B* 44°, and *C* 58°. *AD* is drawn perpendicular to *BC*. To construct a similar triangle of height 2″, from *AD* cut off *AX* 2″ long and through *X* draw *KXL* parallel to *BC*.

24. Construct an equilateral triangle of altitude 69 mm.

A series of parallel straight lines, at equal intervals, may be drawn by using a set-square and the protractor. Place the hypotenuse of the set-square against the protractor, which must be kept fixed. Slide the set-square along the protractor and rule lines as the 'dagger' is placed opposite successive divisions of the scale.

(If the longer edge of a '60' set-square is used for ruling lines, when the dagger is placed opposite successive divisions, find by measurement the intervals between the parallel straight lines.)

25. Draw half a dozen parallel straight lines at 5 mm. intervals.

26. Draw five parallel straight lines at $\frac{1}{4}''$ intervals.

27. Draw an equilateral triangle with sides 3″, and inside it draw a similar figure with its sides ·2″ from those of the original triangle.

28. Draw a rectangle 6 cm. by 4 cm., and divide it up into 25 similar rectangles.

EXERCISE XXII.

AREAS. (I.)

1. Draw two lines AX and AY at right angles. From AX cut off AB 4″ long, and from AY cut off AC 3″ long. Mark off AB and AC in inches with your dividers.

At every inch in AB draw a line parallel to AC, and at every inch in AC draw a line parallel to AB. [These parallels may be drawn with a set-square.] A rectangle $ABDC$ has been obtained.

What is the shape and area of each of the small figures into which it has been divided, and what is the area of the rectangle?

2. By the same method as in question 1, draw a rectangle 9 cm. long and 7 cm. broad, and divide it up into square centimetres. Count the number of sq. cm. in the rectangle.

3. Draw a square with sides 1 dm. By drawing two sets of parallels 1 cm. apart, show that 1 dm^2. = 100 cm^2.

4. Draw a square $PQRS$ each of whose sides is 4″. Divide each side into inches, and join the pairs of opposite points thus obtained so as to divide $PQRS$ into square inches.

Let the diagonals PR, QS intersect in T. Count the number of small squares in the triangle PQS, and in the triangle PQT. What are the areas of these triangles?

5. Repeat question 4, but make the sides of the square 5 cm.

6. Draw a rectangle $ABCD$, in which $AB = 3\frac{1}{2}''$ and $AD = 4''$. Along AB and AD mark off points at distances of one inch; through these points draw lines parallel to AD and AB. How many small squares have been formed, and what is the area of each of the remaining figures? What is the area of the rectangle $ABCD$?

If the two sets of parallel lines had been drawn $\frac{1}{2}''$ apart (instead of $1''$ apart), into how many small squares would the rectangle have been divided, and what would have been the area of each of these small squares?

7. Draw two lines at right angles $2\frac{1}{2}''$ long. Mark off each line in inches from their point of intersection, and through each of these points and at the end of each line draw parallels to the other line.

On each of the small figures thus formed write down its area in square inches. What is the area of the whole square?

8. Repeat question 6, taking each of the lines $2\frac{1}{4}''$ long.

9. Draw a square 8 cm. each way, and divide it into four equal squares by joining the middle points of the sides. Divide one of these squares into 4 equal squares in a similar manner; and divide one of these latter squares again into 4 four squares.

On each of the squares in your diagram write its area.

10. Draw a rectangle 8 cm. by 7 cm. What is the area of each of the right-angled triangles into which it is divided by a diagonal?

11. Find the area of the right-angled triangle ABC in which $a = 3{\cdot}5''$, $b = 2{\cdot}4''$, $C = 90°$.

12. Find the area of the right-angled triangle whose sides are 13 cm., 12 cm. and 5 cm.

13. Draw a parallelogram $ABCD$ with sides AB 9 cm. and AD 5 cm., and included angle BAD 50°. Through A and B draw AE and BF perpendicular to AB to meet CD (or CD produced) in E and F.

Compare the sides and angles of the triangles ADE and BCF, and prove that the parallelogram $ABCD$ and the rectangle $ABFE$ are equal in area.

14. As in question 13, construct a rectangle equal in area to a parallelogram which has sides $3{\cdot}6''$ and $2{\cdot}3''$ and angle 72°, and find its area.

15. On AB as base construct a rectangle equal in area to the parallelogram $ABCD$ in which $AB = 2''$, $AD = 3{\cdot}6''$ and $\angle BAD = 35°$.

16. Draw a parallelogram with sides 5 cm. and 10 cm., and angle 150°. Construct rectangles equal in area to the parallelogram first on one of the short sides as base, and then on one of the long sides as base. Calculate the area in each case.

17. What lengths would you measure to determine the area of a parallelogram? Express its area in terms of these lengths.

Use this expression to find the area of a rhombus $ABCD$, formed of two equal isosceles triangles ABD and CBD of sides 5 cm., 5 cm. and 8 cm. Check your result by adding together the areas of the four right-angled triangles into which the rhombus is divided by its diagonals.

18. Draw a triangle ABC in which $a = 9{\cdot}6$ cm., $b = 7$ cm., $c = 5{\cdot}8$ cm. Through B and C draw BE and CF at right angles to BC, and through A draw a line EAF parallel to BC. From A let fall a perpendicular AD on BC.

What fraction is the triangle ABD of the rectangle $EADB$, ACD of $FADC$ and ABC of $BCFE$? Calculate the areas of these three triangles.

19. Repeat the working of question 18 for the triangle PQR in which $QR = 10{\cdot}5$ cm., $RP = 7{\cdot}5$ cm. and $PQ = 5$ cm., taking one of the shorter sides as base.

20. Draw a triangle ABC having a 2·8″, b 2·5″ and c 1·7″. Through the vertex A draw a line XAY parallel to the base BC; and draw BE and CF perpendicular to BC and meeting XY in E and F.

Find the area of the rectangle $BCFE$, and hence write down the area of the triangle ABC.

21. Use the method of question 20 to find the area of the following triangles:

(i) $a = 3{\cdot}6''$, $b = 2{\cdot}5''$, $c = 2{\cdot}9''$.

(ii) $a = 7$ cm., $b = 7{\cdot}5$ cm., $C = 55°$.

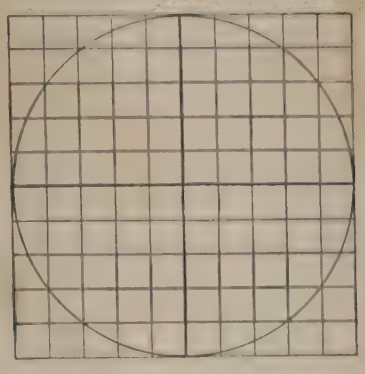
(iii) $a = 10$ cm., $B = 42°$, $C = 58°$.

22. Express the area of any triangle ABC in terms of the base BC and the height AD (the perpendicular on the base). Use this expression to find the area of the triangle in which a is 10 cm., b 8 cm. and c 6 cm. (taking BC as the base). Check your result by taking AB as the base, then AC is the height, as A is 90°.

Any *rectilineal* figure can be divided into triangles, and its area obtained by adding together the areas of these triangles: another method is given in Ex. XXVI.

The area of a *curvilinear* figure cannot be determined *exactly* without a knowledge of advanced mathematics; but a *rough* result may be obtained by dividing it up into small squares and counting the number of squares contained within it (as in the following example).

To find roughly the area of a circle of radius 5 cm.:



Draw two diameters at right angles, and mark them off in centimetres from the centre: through these points on each diameter draw lines parallel to the other. What is the area of each of the small squares?

Find the area of the circle by counting the number of complete squares inside the circle, and estimating the area of the remainder. A more exact result could be obtained by drawing the parallels nearer together.

EXERCISE XXIII.

AREAS. (II.)

1. Calculate the areas of the following six triangles:
   - (i) Sides 12·5 cm., 12 cm. and 9·5 cm.
   - (ii) Sides 4·1″, 3·8″ and contained angle 70°.
   - (iii) Sides 11·9 cm., 8·9 cm. and contained angle 30°.
   - (iv) Sides 3·25″, 3″ and 1·25″.
   - (v) Sides 7 cm. and 9 cm. and included angle 90°.
   - (vi) Base 5″ and base angles 43° and 76°.

2. Calculate the areas of each of the following six triangles *ABC*. Draw a perpendicular from each angular point to the opposite side, and calculate the area by considering each side in turn as base. Tabulate your results under the headings:—base—height—area, and take the average of the *three* values of the area.

(i) $a = 4''$, $b = 4{\cdot}5''$, $c = 3{\cdot}5''$.
(ii) $A = 55°$, $b = 12{\cdot}1$ cm., $c = 9{\cdot}9$ cm.
(iii) $A = 40°$, $B = 62°$, $c = 4{\cdot}4''$.
(iv) $a = 89$ mm., $A = 61°$, $B = 43°$.
(v) $a = 3{\cdot}7''$, $b = 2{\cdot}8''$, $C = 122°$.
(vi) $a = 5{\cdot}5$ cm., $B = 41°$, $C = 110°$.

3. In a circle of radius 5 cm. describe a regular hexagon. Divide it into six equal triangles and find the area of one of them. What is the area of the hexagon?

4. In a circle of radius $1{\cdot}8''$ describe an octagon by drawing two diameters at right angles and bisecting the angles between them. Find the area of the octagon.

5. Calculate the areas of the following six parallelograms:
(i) Sides 10·4 cm. and 6·3 cm. and one angle 63°.
(ii) Sides $4{\cdot}1''$ and $2{\cdot}4''$ and one angle 107°.
(iii) All sides 7·6 cm. and one angle 114°.
(iv) Sides $3{\cdot}1''$ and $1{\cdot}9''$ and one diagonal $4{\cdot}4''$.
(v) Sides 6·2 cm. and 7·4 cm. and one diagonal 11·4 cm.
(vi) All sides $3{\cdot}5''$ and one diagonal $4{\cdot}2''$.

6. Calculate the areas of the following four trapezia $ABCD$ (having the sides AB and DC parallel):
(i) AB 10 cm., AD 5 cm., DC 4 cm. and $\angle A$ 90°.
(ii) AB $3{\cdot}7''$, AD $1{\cdot}4''$, DC $2{\cdot}3''$ and $\angle A$ 90°.
(iii) AB 12 cm., AD 4 cm., DC 8 cm. and $\angle A$ 60°.
(iv) AB $3''$, AD $2''$, DC $1{\cdot}5''$ and $\angle A$ 105°.

7. Draw the following quadrilaterals, and calculate their areas by dividing each of them into two triangles.

N.B.—Draw a rough figure first in each case, and on it mark the given lengths and angles.

(i) $ABCD$, in which AB is $2{\cdot}8''$, BC $3''$, CD $3{\cdot}4''$, DA $2{\cdot}4''$ and the diagonal BD $4''$.
(ii) $EFGH$, in which EF is 6·5 cm., FG 6 cm., GH 11 cm., HE 7·5 cm. and the diagonal EG 10·5 cm.
(iii) $KLMN$, given the diagonal LN $4{\cdot}3''$ and the angles KNL 40°, KLN 30°, MNL 75° and MLN 55°.

(iv) $PQRS$, in which $PQ = 12{\cdot}8$ cm., $QS = 13{\cdot}4$ cm., $SP = 6{\cdot}4$ cm., $SR = 8{\cdot}8$ cm. and $\angle RSQ = 63°$.

(v) $WXYZ$, in which $WX = 4{\cdot}1''$, $WY = 4{\cdot}6''$, $YX = 3{\cdot}5''$, $\angle ZWX = 81°$ and $\angle ZXW = 33°$.

EXERCISE XXIV.

SQUARES AND RECTANGLES.

1. On a base 4″ long construct a rectangle of height 3″. What is its area?

2. Draw a rectangle whose sides are 9·5 cm. and 8 cm., and find its area in (i) sq. cm., (ii) sq. mm.

3. On a base 75 mm. long construct a square. What is its area in (i) sq. mm., (ii) sq. cm.?

4. Draw a square whose sides are 3·8″. Measure its diagonals.

5. Make a square with sides 1″. Measure the sides in centimetres and calculate its area in sq. cm.

How many sq. centimetres = 1 sq. inch?

6. Draw a rectangle 3″ by 4″. Measure its sides in cm., and give its area in sq. in. and in sq. cm. From these two values of its area calculate how many sq. cm. = 1 sq. in. Why is this result more accurate than that obtained in the previous question?

7. Draw a square with sides 1 dm. How many sq. cm. would it contain?

Measure a side of the square in inches, and calculate its area in sq. in. Hence express 1 sq. cm. in sq. in.

8. Draw a rectangle with sides 12 cm. and 10 cm. Measure the sides in inches, and calculate its area in sq. in. From this result express 1 cm². in sq. in.

9. Along a straight line AX measure off AB 4″ and AC 10 cm. On AB and AC construct squares. From your figure deduce a *rough* relation between sq. cm. and sq. in.

10. Construct a rectangle $ABCD$ with sides AB 4″ and AD 3″ by the following method:

Draw two lines AX, AY at right angles: from AX cut off AB 4″, and from AY cut off AD 3″. With centre B and radius 3″, and with centre D and radius 4″, describe arcs cutting one another at C.

Prove that the figure $ABCD$ is a rectangle.

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11. By the method of question 10, construct a rectangle with sides 40 mm. and 96 mm. Measure its diagonals.

12. By the method of question 10, draw a square with sides 7·2 cm. Let the diagonals intersect at *O*. From *O* let fall a perpendicular *ON* on to one of the sides. With *O* as centre and *ON* as radius describe a circle.

13. Construct a rectangle with sides 4·2″ and 3·3″. Join the middle points of adjacent sides, and prove that the figure so formed is a rhombus. What is the area of this rhombus?

N.B.—Prove that the angle in a semicircle is a right angle, and use this fact in the following questions.

14. In a circle of *any* radius draw two diameters at right angles and join their extremities. Prove that the figure so formed is a square, and check your proof by measurements.

15. Draw a line *AB* 4·3″. Bisect *AB* at right angles at *O* by the line *XOY*. With centre *O* and *AB* as diameter describe a circle cutting *XY* in *C* and *D*. Compare the lengths *AC*, *CB*, *BD* and *DA*. Is the figure *ACBD* a square or a rhombus?

16. Construct a square on a diagonal of 12 cm. What is its area?

17. Construct a square with diagonal 71 mm. Measure its sides.

18. Draw any circle and in it place *any* two diameters *AOB* and *COD* (not necessarily at right angles). Prove that the figure *ACBD* is a rectangle, and compare its opposite sides with your dividers.

19. Draw *AB* 5·1″ and bisect it at *O*. With *O* as centre describe a circle to pass through *A* and *B*. With *A* and *B* as centres and radii 2·4″ describe arcs cutting the circle in *C* and *D* on opposite sides of *AB*. Join and measure *AD* and *BC*.

20. Construct a rectangle, with a diagonal 10 cm. and one side 8 cm. Measure the other sides, and find its area.

21. Construct a rectangle with diagonal 5·2″ and two sides 2″. Measure the other two sides.

22. Draw a circle centre *O* and radius 4·7 cm. Draw a diameter *AB*. With centre *A* and radius 3·5 cm. describe an arc cutting the circle at *C*. Draw the diameter *CD*. Compare *BD* and *AC*, and prove them equal, and prove that *ACBD* is a rectangle.

EXERCISE XXV.

RIGHT-ANGLED TRIANGLES (THEOREM OF PYTHAGORAS).

1. Draw a triangle with sides 3 cm. and 4 cm. and contained angle 90°. On each side describe squares externally, and compare the area of the square on the hypotenuse with the sum of the squares on the sides.

2. Draw *any* right-angled triangle and on its sides describe squares. Find the area of each, and compare the area of the largest with the sum of the two others.

3. Draw a triangle with B 90°, a 1″, c 2″. Find the area of the square on AC (i) by measurement, (ii) by calculation.

4. PQR is a triangle with Q a right angle and PQ and QR each 5 cm. Find the area of the square on PR and check by calculation.

5. ABC is a triangle with C a right angle and a 7 cm. and b 6 cm. Calculate the length of c and check by measurement.

6. In the right-angled triangle XYZ, the hypotenuse $YZ = 2{\cdot}9''$ and the side $XY = 2''$.

Calculate the length of the side XZ and verify your result by measurement.

7. In the triangle ABC, $a = 10$ cm., $b = 7$ cm., $A = 90°$. Find c by calculation and by measurement.

8. A ladder 41 feet long is placed against a wall, with its foot 9 feet from the bottom of the wall. Calculate the height to which the ladder reaches. Draw a diagram and verify your result.

9. Find the length of the diagonal of a football field 120 yds. long and 80 yds. wide. Check by measurement.

10. A wire is stretched across a road 32 feet wide, between the tops of two poles 12 ft. and 27 ft. high; find its length from a diagram, and check by calculation.

11. Find, by calculation, which of the following triangles are right-angled and check your result by figures drawn to scale:

(i) Sides 6 cm., 8 cm., 10 cm.

(ii) Sides 10 cm., 6 cm., 12 cm.

(iii) Sides 7 miles, 12 miles, 13 miles.

(iv) Sides 41 yards, 40 yds., 9 yds.

(v) Sides 9″, 7″, 11½″.

(vi) Sides 7 ft. 6 in., 8 ft. 6 in., 4 ft.

12. Draw a triangle with a 2·1″, b 2″ and C a right angle. On each side, externally, describe equilateral triangles. Compare the area of the equilateral triangle on AB with the sum of those on BC and AC.

13. Draw a triangle with A 90°, b 12 cm., c 5 cm. On each side describe similar isosceles triangles with base angles 40°. Find the area of each isosceles triangle, and compare the area of that on BC with the sum of those on AB and AC.

EXERCISE XXVI.

AREAS. (III.) (MISCELLANEOUS THEOREMS.)

To construct a triangle equal in area to a given rectilineal figure.

1. Draw a quadrilateral $ABCD$ with AB as base; given AB 10·4 cm., BC 7·8 cm., CD 5 cm., DA 12 cm. and diagonal AC 13 cm. Through C draw CG parallel to DB and meeting AB produced in G. Join DG. Draw DK perpendicular to AB.

Find the area of the triangle DAG.

2. Draw *any* rectilineal figure $ABCD$, and prove that it is equal in area to the triangle DAG (constructed as in question 1). Check your result by finding the area of DAG and comparing it with the area of the quadrilateral found from the triangles ABD and CBD.

Find the area of each of the four following rectilineal figures by constructing a triangle equal to it in area:

3. $ABCD$, in which $AB = 2{\cdot}7''$, $AD = 2{\cdot}1''$, $BC = 3{\cdot}1''$, $\angle BAD = 105°$ and $\angle ABC = 65°$.

4. $PQRS$, in which $PQ = 72$ mm., $QR = 54$ mm., $\angle QPS = 60°$, $\angle PQR = 90°$ and $\angle PQS = 75°$.

5. $ABCDE$, in which the points A, B, C, D, E lie on a circle of radius 2″ and centre O; given $\angle AOB = 50°$, $\angle AOC = 130°$, $\angle AOD = 230°$ and $\angle AOE = 310°$. In this figure take A as the highest point of the circle.

6. $PQRST$, in which all the points lie on a circle of radius 5 cm., and $PQ = PT = 6$ cm., $PR = PS = 9$ cm. In this figure take P as the highest point of the circle.

7. Find the area of a field in the form of a quadrilateral $ABCD$, in which the sides are AB 300 yards, BC 330 yds., CD 180 yds., DA 280 yds. and the diagonal AC is 400 yds.

8. Find the area of a quadrilateral field $ABCD$, in which AB is 350 yards, BC 270 yds., CD 210 yds., DA 400 yds and $\angle ABC$ is 80°.

To construct a parallelogram (or triangle) equal in area to a given triangle and having an angle equal to a given angle.

9. Construct a parallelogram having one of its angles 45° and equal in area to an equilateral triangle of side 3·2″.

10. Draw a triangle of sides 11 cm., 9 cm. and 8 cm., and describe a parallelogram equal in area to this triangle and having an angle of 75°.

11. Construct a rectangle equal in area to a triangle which has a base 4·5″ and base angles 55° and 70°.

12. Construct a rectangle equal in area to an isosceles triangle of base 7 cm. and equal sides 9 cm. Also draw a right-angled triangle equal in area to the given triangle, and on the same base.

13. Draw a triangle ABC, in which $a = 9{\cdot}5$ cm., $c = 8$ cm. and $B = 60°$. On the same base describe three triangles DBC, EBC and FBC of the same area, but having the angles (i) DBC 40°, (ii) EBC 75°, (iii) FBC 115°.

14. Describe an isosceles triangle equal in area to the triangle ABC, in which $a = 2{\cdot}3''$, $b = 3{\cdot}2''$ and $B = 48°$.

On a given base to construct a triangle (or parallelogram) equal in area to a given triangle and having an angle equal to a given angle.

15. ABC is a triangle with a 4″, B 45°, C 80°. On a base 2·7″ make an equal triangle with one angle 50°.

16. Draw a triangle with sides 10·2 cm., 7 cm. and 11 cm. On a base 8 cm. make a parallelogram of equal area with one angle 60°.

17. Construct a triangle with two sides 7 cm. and 10 cm. and included angle 122°. On a base 11 cm. make an equal triangle with one angle 55°. Verify by finding the area of each triangle.

18. ABC is a triangle with a 4″, b 4·2″, c 2·8″. On a base 3″ describe an isosceles triangle of equal area. Measure its altitude.

19. On a base of 7 cm. construct a rectangle equal in area to a triangle ABC, in which $a = b = 8{\cdot}5$ cm., $C = 105°$. Calculate its area.

20. On a base 3″ long describe a rectangle equal in area to an equilateral triangle of side 4·2″. Calculate its area.

EXERCISE XXVII.

DIVISION OF STRAIGHT LINES INTO EQUAL PARTS, AND IN A GIVEN RATIO.

1. Draw a straight line 5″ long. Divide it into 3 equal parts. Measure the parts in millimetres.

2. Draw a straight line 4″ long. Trisect it. Measure the parts in mm.

3. Draw a straight line 14·4 cm. long. Divide it into 5 equal parts and trisect one of the end parts.

4. Divide a straight line 5·2″ long into 3 equal parts and divide one of them into 4 equal parts.

5. Divide a straight line 4″ long into two parts so that one part is double the other.

6. Divide a straight line 13·5 cm. long into two parts so that one part is three times the other. Measure the longer part. Check your answer.

7. Divide a line AH 5·6″ into 7 equal parts; letter the points B, C, D etc. Which of these points divides the line in the ratio of 5 : 2?

8. Divide a straight line 4·7″ long in the ratio 2 : 3. Measure the shorter part in mm.

9. Divide a straight line 13·5 cm. long in the ratio 4 : 5.

10. Divide a straight line 13·5 cm. long in the proportion of 2 : 3 : 4. Measure the largest part. Test your answer by calculation.

11. Draw a triangle ABC, making a 4·3″, c 3·9″, $B = 55°$. Divide BC into three equal parts BD, DE and EC; and AC into three equal parts AF, FG and GC. Join and measure DF and EG.

12. On a base BC 3″ long draw *any* triangle ABC. From AB cut off AD two-thirds of AB, and from AC cut off AE two-thirds of AC. Join and measure DE. What fraction is it of BC? Compare the angles ADE and ABC.

13. Draw pattern No. 23.

14. Construct a square $ABCD$ with sides 3″ long. Find points G, H, K, L on the sides AB, BC, CD, DA respectively such that $AG = \frac{1}{3}AB$, and

BH, *CK*, *DL* and *AG* all equal. Prove that *GHKL* is a square and verify by measuring its sides and angles.

15. Draw a square *ABCD* with sides 6 cm. Take a point *O* outside the square so that $AO = 7$ cm. and $OB = 8$ cm. Join *OA*, *OB*, *OC* and *OD*, and on them take points *P*, *Q*, *R* and *S* such that $OP = \frac{1}{3}OA$, $OQ = \frac{1}{3}OB$, etc.

Show that *PQRS* is a square whose area is one-ninth of the area of *ABCD*.

EXERCISE XXVIII.

SCALES.

1. Draw a scale, of miles and furlongs, sufficiently long to measure lengths up to 5 miles, for a map in which a distance of 2 miles is represented by 2·3″. Proceed as follows :

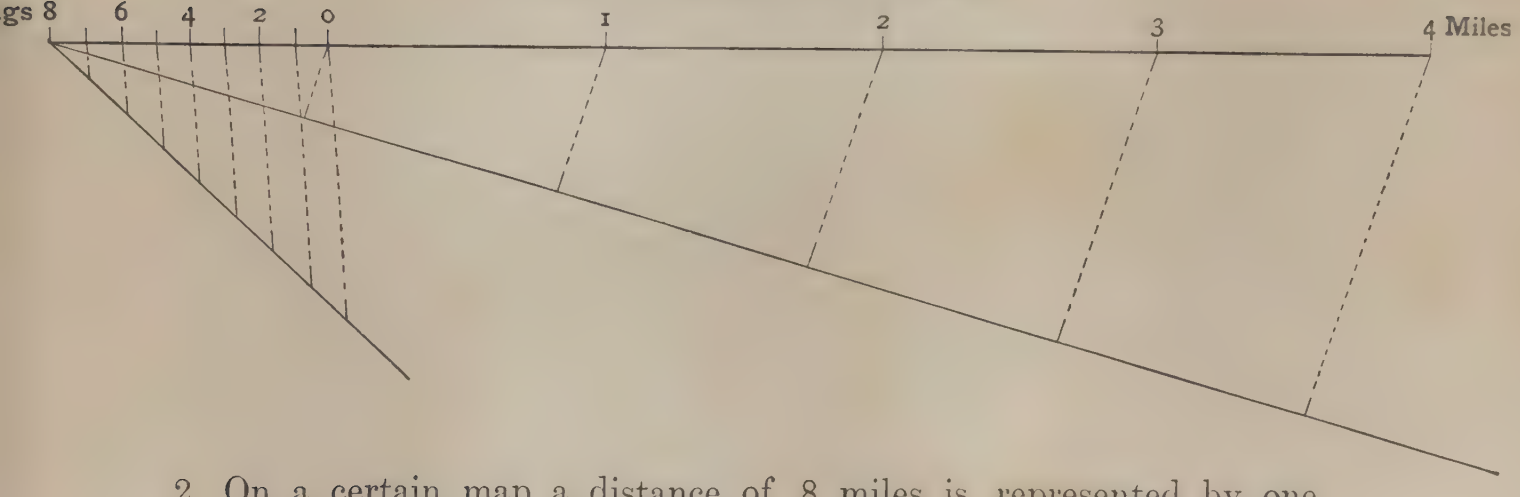
(i) Calculate the total length of the scale ;
2 miles is represented by 2·3″.
∴ 5 miles ,, ,, $\frac{5}{2} \times 2{\cdot}3'' = 5{\cdot}75''$.

(ii) Draw a line of this length (5·75″), and divide it into five equal parts, so as to show single miles.

(iii) Divide the left hand part again into eight equal parts, so as to show single furlongs.

(iv) Figure and describe the scale as below :

Scale of Miles and Furlongs.



2. On a certain map a distance of 8 miles is represented by one inch. Draw a scale sufficiently long to show 40 miles ; divide it into four equal parts, and the left hand part divide again into ten equal parts. Figure and describe the scale.

3. A model is made of an engine in which 1 yard is represented by ·9″. Draw a scale to show 6 yards, and divide it so as to measure yards and feet. From your scale draw a line to represent 4 yds. 2 ft.

4. On a plan 25 yards occupy 3·25″. Draw a scale to show 40 yards, and divide it to read single yards.

5. In a map 1·26″ represents 7 miles. Draw a scale for the map to show 30 miles and divide it to read single miles.

6. The scale of a map is 5·4″ long and represents 50 miles. Divide it to measure single miles. Draw a line to represent 33 miles.

7. On a map 14 Km. is represented by 6·3 cm. Draw a scale of kilometres to show 30. Draw a line to represent 21 Km.

8. Draw a scale to measure Km. and Hm. for a map in which 3·7 Km. occupy 11·1 cm., and show 4 Km.

9. On the plan of a building a distance of 4 yards is represented by 2·28″. Draw a scale, of yards and feet, to show 10 yards.

10. A map is made of a garden and on it 3 chains 75 links is represented by 4·5″. Draw a scale for the map to show 4 chains; and divide it to read chains, and distances of ten links.

Draw scales for the following maps in which lengths are reduced :

11. One-thirtieth. Show 5 yards and single feet.

12. One-fortieth. Show 6 yards and single feet.

13. $\frac{1}{300}$ Show 50 yards and single yards.

14. $\frac{1}{60000}$ Show 5 miles and single furlongs.

15. $\frac{1}{20000}$ Show 3 kilometres and single hectometres.

16. $\frac{1}{4000}$ Show 6 Hm. and single Dm.

17. $\frac{1}{3300}$ Show 4 chains; divide to read to 10 links.

18. $\frac{1}{800}$ Show 5 chains; divide to read to 10 links.

MISCELLANEOUS EXAMPLES.

1. Draw a straight line 5″ long. Measure it in millimetres. Calculate the number of mm. in 1″.

2. Draw 5 concentric circles with radii 1, 2, 3, 4 and 5 cm.

3. Draw a triangle ABC such that $AB = 2\frac{1}{2}''$, $BC = 2''$, $CA = 1\frac{1}{2}''$. With centres A, B, C describe circles with radii 1″, $1\frac{1}{2}''$ and $\frac{1}{2}''$ respectively.

4. Divide a straight line 3·4″ long into four equal parts. Measure the first part in mm.

5. Describe a semicircle with centre C and radius 1·9″. With the ends, A and B of its diameter, as centres describe two circles to pass through C and to cut the semicircle at D and E. Join DA, DC, DB, DE and EA. State the species of each of the triangles ADC, ADB, ADE from its appearance.

6. Pattern No. 1.

7. Draw a line 75 mm. in length; write down its length in inches, and thence calculate the number of inches in a metre.

8. On both sides of a straight line 2·3″ long describe equilateral triangles. What is the shape of the quadrilateral so formed? Draw and measure its other diagonal.

9. Divide a straight line 13·3 cm. long into four equal parts. Measure one part in inches.

10. With centre C and radius 6·8 cm. describe a circle. Draw a diameter AB. Find two points D and E on one semicircle such that $AD = 12$ cm., $AE = 8·5$ cm. Join CD, CE, ED, etc. State the species of each of the triangles ADE, ADC, AEC, AEB, CED, AED from its appearance.

11. A man walks along a straight road from A to B 2 miles, when he finds that he has lost his purse. He goes back half a mile and finds it, and then walks (through B) 4 miles to C. Draw a diagram showing his walk and the places at which he stopped.

12. Inscribe a regular hexagon in a circle, radius 53 mm.

13. A man walks North 3 miles, then South 2·5 miles, again N. 2·4 miles and S. 1·8 miles. Mark the points at which he starts, turns and stops. How far is he finally from his starting point? Check your answer by calculation.

14. Draw a straight line AX, terminated at A. On it take points P, Q, R so that $AP = 2{\cdot}3''$, $PQ = 1{\cdot}1''$, $QR = 2{\cdot}3''$. Measure and calculate AR.

15. Draw any two straight lines BA and AC meeting at A. With centre A and radii $1''$, $2''$, $3''$ describe circles cutting BA at P, Q, R and AC at X, Y, Z respectively. Join and compare PX, QY, RZ.

16. On a straight line $3''$, as base, and on opposite sides of it describe isosceles triangles with equal sides $2{\cdot}5''$. What is the shape of the quadrilateral so formed? Draw and measure its other diagonal.

17. Draw a triangle ABC in which $AB = 6$ cm., $BC = 73$ mm. and $CA = 1$ dm. Bisect BC at D. Join and measure AD.

18. Pattern No. 2.

19. A man walks N.E. 11 Km., then S.W. 9·2 Km., again N.E. 7·3 Km., and finally S.W. 13·1 Km. Mark the points at which he starts, turns and stops. How far and in what direction is he from his starting point? Check your answer.

20. Draw a triangle with sides 3 cm., 3·7 cm. and 43 mm. On its sides externally describe equilateral triangles. Join the vertex of each equilateral triangle to the opposite angular point of the triangle.

21. A triangular park ABC has sides AB 310 yds., BC 420 yds. and CA 270 yds. In it a house H is situated exactly 220 yds. from each of the lodges B and C. How far is it from the lodge A?

22. Obtain a length of exactly $2{\cdot}35''$ by bisecting a straight line.

23. In a circle, of diameter $2''$, place a regular hexagon. On each side, finding the middle point by measurement, describe a semicircle externally.

24. Draw a straight line 1 dm. long. Measure it in inches and calculate the number of inches in a metre.

25. A man walks West 10 miles, then East 3·5 miles, again W. 2·7 miles and finally E. 11·3 miles. Mark the points at which he starts, turns and stops. How far and in what direction is he from his starting point? Scale 1 cm. to 1 mile. Check by calculation.

26. On a base 2″ long and on opposite sides of it describe two isosceles triangles with sides 3·2″ and 1·3″. Measure the other diagonal of the quadrilateral so formed.

27. Describe a circle with diameter *AB* 13 cm. With centre *A* and radius 5 cm. describe an arc cutting one semicircle at *C*, and with centre *B* and the same radius describe an arc cutting the other semicircle at *D*. Join *AC*, *CB*, *BD*, *DA*. Measure the sides of the figure *ACBD*. What is its shape?

28. Pattern No. 4.

29. Draw the triangle *ABC* with *BC* 3·1″, *CA* 2·7″ and *AB* 3″. Bisect *BC* at *D*. Join and measure *AD*.

30. Describe a circle, with radius 5 cm., and draw the six radii to the angular points of a regular hexagon inscribed in it.

Bisect the radii and on them as diameters describe circles.

31. Construct a triangle with sides 8 cm., 9·3 cm. and 10·2 cm. Measure the angles.

32. A man walks 5 miles in a straight line, then turns to the right 43° and walks 4 miles straight on. He then turns again to the right 74° and walks on 3 miles. How far is he now from his starting point?

33. Draw an equilateral triangle with sides 2″, and on its sides externally describe isosceles triangles with sides 1·3″. With the vertices of these isosceles triangles as centres and radii 1·3″ describe circles.

34. A triangular park has sides 1300 m., 950 m. and 1170 m. A lodge is built at each angular point, and straight drives lead from the lodges to the house, which is situated at a distance of 750 metres from the lodges at the ends of the longest side of the park. Find the distance from the house to the third lodge.

35. Draw a circle with radius 3 cm., and inscribe a regular hexagon. On each side of the hexagon, and external to it, describe isosceles triangles with equal sides 4 cm.

36. Construct a triangle ABC, given AB 7·5 cm., BC 7·8 cm. and AC 6·9 cm. Bisect AB at D and AC at E. Measure DE.

37. Draw an equilateral triangle with sides 10 cm., and on each side, internally, describe isosceles triangles with sides 5·4 cm.

38. Make a triangle with base 3·4″ and base angles each 54°. Measure its remaining sides.

39. Construct a hexagon in a circle of radius 2″. Draw three radii so as to divide it into three equal rhombi.

40. Draw an equilateral triangle with sides 4·7 cm. Measure and write down the size of its angles. Bisect the sides and on them, externally, describe semicircles.

41. A man walks 8 miles due E., then 6 miles N.W., and lastly 2 miles N. Draw a diagram of his journey. [Scale 2 miles to the inch.] How far is he finally from his starting point?

42. On a base 2″ describe an isosceles triangle with sides 3·4″. Measure and write down the size of the base angles. Produce the equal sides and measure the angles on the other side of the base.

43. Draw *any* five straight lines meeting at a point, and measure the five angles so formed. What is the sum of these angles?

44. Find three points at equal distances of 4 cm. from each other. With these points as centres describe circles of radii 3 cm. Join the points of intersection of each pair of circles.

45. A railway bridge 350 yards long crosses a river obliquely, making an angle of 70° with the direction of the river. Find the breadth of the river from a diagram drawn to scale.

46. PQR is an isosceles triangle in which $PQ = PR = 7$ cm. and $\angle QPR = 50°$. The side QP is produced to S, making PS equal to PQ. Measure the angle QRS.

47. Describe a triangle with sides 4·3″, 5·2″ and 3·7″. Find the middle points of the sides, and join these points to the opposite angular points of the triangle.

48. In a circle of radius 1·3″, describe a regular hexagon; produce each side both ways to form a star-shaped figure, and describe a circle to pass through the corners of this figure. What is the radius of this circle?

49. Draw a triangle ABC with sides 7 cm., 6 cm. and 3 cm. With A, B, C as centres describe circles of radii 4 cm., and join the points of intersection of each pair of circles.

50. Two ships take up their positions 1000 yards apart to shell a battery. From each ship the angle between the battery and the other ship is observed to be 75°. How far are the ships from the battery? [Scale 500 yds. to the inch.]

51. Draw a large irregular pentagon. Measure all its interior angles and find their sum.

52. On opposite sides of the straight line AC (4·8″ long) describe triangles BAC, DAC in which $BA = 2·7''$, $BC = 2·3''$, $CD = 4·2''$, $DA = 3·8''$. Bisect AB, BC, CD, DA respectively at K, L, M, N, and join KL, LM, MN, NK. Measure and compare the lengths of KL and MN, also of LM and NK.

53. Draw a circle of radius 2·2″, and on its circumference mark the angular points of a regular hexagon $ABCDEF$. Join alternate points to form two equilateral triangles. What is the length of their sides?

54. Draw a triangle with sides 90 mm., 72 mm. and 54 mm. With the two smaller sides as diameters describe circles. Do these circles cut the longest side of the triangle at the same point?

55. On a base 5·2 cm. and on opposite sides of it describe isosceles triangles with sides 7·3 cm. What is the shape of the quadrilateral so formed? Bisect each of its sides and on them as diameters describe semicircles externally.

56. Draw *any* large irregular hexagon. Measure all the interior angles. Find their sum.

57. PQR is a triangle in which $PQ = 10·5$ cm., $\angle P = 25°$ and $\angle Q = 104°$. Measure QR.

58. A road runs from A to B east 12 Km., turns to the south, and goes from B to C 5 Km. Draw a diagram of the road and measure the distance from A to C direct.

59. Pattern No. 3.

60. Inscribe, in a circle radius 2·3″, a regular hexagon *ABCDEF*. Join *AC* and *AD*. Measure the angles of the triangle *ACD*.

61. On a diameter *AOB* describe a semicircle with centre *O* and radius 2·5″. Make the angles *AOP* 24°, *AOQ* 43°, *AOR* 62°, *AOS* 81°, *AOT* 100° the points *P*, *Q*, *R*, *S*, *T* being on the circumference. Measure the chords *PQ*, *QR*, *RS*, *ST* and prove them equal.

62. On a base 5·3 cm., and on opposite sides of it, describe two isosceles triangles with sides 6·4 cm. What is the quadrilateral so formed? Prove its opposite angles equal, and verify by measurement.

63. Draw angles (i) of 53°, (ii) of 157°. Bisect each by construction and measure the halves.

64. A tower *CD* is surrounded by houses. A man measures a line *AB* 150 feet long down a street which points directly towards the tower. If the angles *CAD*, *CBD* be 18° and 26°, what is the height of the tower?

65. Draw a straight line *AB* 3·1″ long; in it take a point *C* 1·5″ from *A*. At *C* draw a line perpendicular to *AB*.

66. Draw a straight line *AB* 10 cm. long. Find a point 7 cm. from *A* and 5 cm. from *B*. Through this point draw a line perpendicular to *AB*.

67. Describe a circle with radius 2″ and draw 5 radii making, with each other, angles 27°, 43°, 82°, 119°. Measure and calculate the remaining angle.

68. Draw a straight line 3″ long; at its extremities, *on both sides of it*, make angles 35°. Prove that the sides of the resulting quadrilateral are all equal. Check by measurement.

69. Draw an angle *BAC* of 47°. Make *AB* 7·5 cm. At *B* erect a perpendicular to *AB* meeting *AC* at *K*. Measure *KB*.

70. Make an angle of 125° and divide it into four equal parts.

71. Make an angle *BAC* 37° (with arms at least 10 cm. long). On each arm take a point 72 mm. from the angle and from it drop a perpendicular on the other arm. Measure the perpendiculars and prove them equal.

72. Pattern No. 5.

73. With your protractor make the following angles adjacent to each other, 37°, 43°, 17°, 53°. Measure the angle formed by the sum of these. Check your result by calculation.

74. ABC is a triangle in which $a = 3{\cdot}5''$, $\angle C = 61°$, $\angle B = 46°$. Measure AB.

75. Draw AB 2·3″ long, and AC and BD at right angles to AB respectively 2·1″ and 2·5″. Bisect AB at E; and draw EF at right angles to AB, cutting CD at F. Measure EF.

76. Make an angle of 65°, bisect it and find also its fourth part.

77. ABC is a triangle in which $a = 5''$, $b = 4''$ and $c = 3''$. From A let fall a perpendicular AD on BC. Measure AD, BD, DC.

78. The wire stay of a telegraph pole is fastened to the top of the pole and to a point in the ground 2 metres from its foot. The angle it makes with the ground is 75°. Find the length of the wire rope and the height of the pole.

79. Make angles, 113°, 27°, 39°, 102° adjacent to each other, at a point. Measure the remaining angle. What is the total sum of the angles at the point?

80. Draw a straight line 8 cm. long; at its extremities, *on both sides of it*, make angles 45°. What is the shape of the resulting quadrilateral?

81. On the circumference of a circle, radius 5·3 cm., any three points A, B, C are taken. The angle BAC is bisected by AD meeting the circumference at D. Join BD, DC. Measure and compare them.

82. ABC is a triangle with a 12 cm., B 39°, C 57°. In BC take points D and E so that $BD = 4$ cm. and $CE = 2$ cm. At D and E erect perpendiculars to BC cutting AB and AC at K and L respectively. Measure DK and EL.

83. Draw AB 3·5″ long and AC, at right angles, 4·3″ long. Join BC. From A drop a perpendicular AD on BC. Measure the angle BAD.

84. Pattern No. 7.

85. XYZ is a triangle in which $YZ = 4{\cdot}4''$, $\angle Y = 22°$, $\angle Z = 91°$. Measure ZX.

86. Draw a straight line AB meeting another straight line KBL at *any* angle. Bisect the angles KBA and ABL by BX and BY. Prove that XBY is a right angle. Check by measurement.

87. Draw a straight line AOB 8 cm. long. Bisect it at right angles by COD. Make CO and OD each 4 cm. Join AC, AD, BC and BD. What is the shape of $ACBD$?

88. Draw AB 3·6″ long, AC at right angles 3″ long. Join BC, and from A draw AD perpendicular to BC. Measure AD and the angle BAD.

89. Two coastguards at points A and B, 1000 yds. apart on a straight coast line, observe a ship C at the same time. They find the angles BAC, ABC to be 67° and 52° respectively. Find, by drawing a diagram, the distance of the ship from the shore. State your scale. Why are observations from more than one point necessary to determine the position of the ship?

90. With centre C and radius 1·8″ describe a circle. At K, the extremity of one of the radii CK, draw KA 2·4″ perpendicular to CK. Join AC cutting the circle at B and D. Measure AB, AC, AD.

91. Draw a triangle ABC with a 10 cm., b 13·2 cm. and C 43°. Draw the bisectors of all the angles to meet the opposite sides.

92. Draw a straight line POR 2·7″ long, bisect it at right angles by QOS. Make OQ and OS each 1″. Join PQ, QR, RS, SP. Prove that $PQRS$ is a rhombus. Check by measuring its sides.

93. Draw a triangle with sides 4·7″, 3·7″ and 3·2″. Bisect all the sides and join all the angular points to the middle points of the sides.

94. A man measures the angle CAD subtended a by tower CD at a point A on the ground 300 feet from its base. If $\angle CAD = 36°$, find the height of the tower.

What error would be made in the height of the tower if the angle measured were only 33°?

95. Draw a line AB 3·8″ long; at A draw AC at right angles, at B make the angle ABC 35°. From A drop a perpendicular on BC. Measure and write down its length.

96. Pattern No. 8.

97. On opposite sides of a base AB 5″ long describe triangles ACB and ADB with AC 3″, BC 4″, AD 4″ and BD 3″. Bisect the angles at C and D and measure the angles which the bisecting lines make with AB.

98. Draw AB 3·5″ and bisect it at C. Draw CD 3″ long at right angles to AB. Join AD and bisect the angle DAC by AE meeting CD at E. Measure AE and $\angle DAE$.

99. Construct the triangle ABC having a 2·8″, b 3·1″, c 4·2″. Let the lines which bisect the sides BC and AC at right angles meet at S. With centre S and radius SC describe a circle.

100. Draw a triangle with sides 10 cm. and 8·7 cm. and included angle 63°. Measure the third side. Bisect all the angles.

101. A man places a 3-foot stick upright on the ground at a distance of 16 ft. from a lamp-post. He finds that the shadow of the stick thrown by the light is 4 ft. long. Find the height of the lamp-post. In your diagram let 1″ represent 5 feet.

102. Draw AB 4″ long and draw BC 3″ long perpendicular to AB; join AC and draw BD perpendicular to AC. Measure CD and $\angle ACB$.

103. Draw a straight line AOB 8·4 cm. long. Bisect it at right angles by COD. Make OC and OD each 4·2 cm. Join AC, CB, BD, DA. Prove that $ACBD$ is a square. Check by measuring its sides and angles.

104. Construct a triangle ABC with a 10·3 cm., b 8·1 cm. and C 72°. Draw the bisectors of the three angles to meet the opposite sides.

105. Construct an equilateral triangle with altitude 5 cm. Measure its sides.

106. Draw AB 8 cm. long and AC perpendicular to it 11 cm. long. Join BC and drop a perpendicular AD on it. Measure AD and the angle BAD.

107. Pattern No. 6.

108. In the triangle ABC, $B = 71°$, $C = 53°$ and $a = 2·9″$. Join the middle points of the sides to the opposite angular points.

109. In a circle, radius 3 cm., draw two diameters at right angles; with their extremities as centres describe four semicircles with radii 3 cm. to cut one another in A, B, C, D.

110. Construct a triangle ABC given b 3·2″, c 2·7″ and A 57°. Measure a and bisect the angles at B and C.

111. Draw an isosceles triangle with base 3″ and altitude 1·9″.

112. Draw a triangle ABC with sides 11 cm., 9 cm. and 13 cm. Bisect two of its angles, and from the point K where the bisectors of these two angles intersect let fall a perpendicular KD on to the longest side. With centre K and radius KD describe a circle.

113. To measure the breadth of a river :—A man places a pole at a point A on one bank exactly opposite to a tree B on the other side. He then walks a distance of 300 yards along the bank to C, so that AC is at right angles to AB. At C he measures the angle BCA to be 53°. What is the breadth of the river?

114. Obtain a length exactly 2·05″ by bisecting a line.

115. Draw a triangle ABC with a 8 cm., B 37°, C 113°. Bisect AB and AC at right angles by straight lines meeting at S. Prove that $SB = SA = SC$. With centre S and radius SA describe a circle.

116. Draw a triangle ABC with c 3″, b 3·7″, a 2·9″. Produce AB to D and AC to E. Bisect the exterior angles DBC, ECB by straight lines meeting at F. Join AF and measure the angles FAB and FAC.

117. Draw a straight line 8·2 cm. long and bisect it at right angles. With the point of bisection as centre and radius 4·1 cm. describe a circle. On each of the four radii as diameter describe a circle.

118. ABC is a triangle in which $a = 87$ mm., $B = 40°$, $C = 65°$. Draw AD perpendicular to BC. Bisect CA and AB at M and N respectively. Measure the angles BAC and NDM.

119. Construct an equilateral triangle, side ·072 m., and with one of its sides as altitude construct another.

120. Pattern No. 9.

121. Draw a triangle ABC with a 3·7″, b 3·2″ and c 2·7″. Bisect the two longest sides at right angles by straight lines meeting at S. Prove that $SA = SB = SC$. Check by describing a circle with centre S and radius SA.

122. How many degrees are there in the base angles of any isosceles triangle, if each is half the vertical angle? On a base 10·5 cm. long describe such a triangle.

123. Construct the triangle ABC having c 2·6″, b 3·7″ and A 55°. Through B and C draw, by construction, lines parallel to the opposite sides meeting at D. Prove that $\angle CDB = \angle CAB$. Check by measurement.

124. Draw a square with sides 2″. Draw the diagonals and join the middle points of adjacent sides. Find by calculation and write down the area of each of the eight small triangles. If the middle points of the inner square be joined another square is formed. Measure its sides.

125. Pattern No. 10.

126. A triangle ABC has C 95°, B 45° and a 45 mm. By copying two angles make a similar triangle with longest side 4″. Measure the remaining sides of this triangle in inches.

127. Draw two lines AB, AC each 3″ long, inclined at an angle of 43°. Bisect the angle BAC by AD. Take E the middle point of AB and through it draw EF parallel to AC to meet AD in F. Prove that the triangle EFA is isosceles.

128. Construct straight lines which make angles of 30°, 60°, 90°, 120°, 150° with OA. Divide the angle of 30° into 3 equal parts by trial to show 10° and 20°.

129. ABC is a triangle in which $c = 7{\cdot}9$ cm., $a = 67$ mm. and $B = 67°$. Let the bisectors of the angles A and C meet at X. From X drop perpendiculars XD, XE, XF on the sides and prove them equal. Verify by describing a circle with centre X and radius XD.

130. A rhombus has sides 2·5″ and one angle 75°. Find its area.

131. Construct a hexagon in a circle of radius 5 cm. From the centre drop a perpendicular on one of the sides. Find the area of the hexagon by dividing it into six equal triangles.

132. With centre A describe a circle of radius 1·5″. At B, any point on the circumference, construct a perpendicular BC, 2″ long, to AB. Join CA cutting the circle at D and E. From B construct BF perpendicular to AC. Produce BF to cut the circle at K. Prove that (i) $BF = FK$ and (ii) $BC = CK$. Check by measurement.

133. A man standing at a point A 350 yards from the foot of a tower BC finds the angle BAC to be 27°. What is the height of the tower? To what extent would the calculated height be increased, or decreased, by an error of 3° in the angle BAC?

134. Pattern No. 11.

135. How many degrees are there in the base angles of any isosceles triangle, if each is double the vertical angle? On a base 3·1″ long, describe such a triangle.

136. Draw a triangle with sides 3″, 2·8″, 3·4″. By copying the angles draw a triangle of the same shape with its longest side 102 mm. Measure the other sides.

137. With base BC 2″ and base angles each 70° construct a triangle ABC. Let the lines which bisect AB and AC at right angles meet in S. Prove that $SA = SB$. With S as centre draw a circle to pass through A, B and C.

138. Describe a triangle ABC in which $a = 3{\cdot}2''$, $B = 90°$, $C = 49°$. Through A draw AD, 1·1″ long, parallel to BC. Calculate the areas of the triangles ABC and DBC.

139. Draw a rectangle 8 cm. long and 5 cm. broad. Divide it into 5 rectangles, each 8 cm. long and 1 cm. broad; and divide one of these latter rectangles into 8 equal squares.

140. Pattern No. 12.

141. Draw an isosceles triangle ABC in which $a = 1{\cdot}9''$, $B = C = 62°$. Produce BA to D making AD equal to AB. Prove that $\angle DCB$ is a right angle, and verify by measurement.

142. Draw a triangle ABC, given a 8·5 cm., b 9·2 cm. and C 71°. Bisect the angles B and C by lines meeting at I. Draw ID perpendicular to BC. With centre I and radius ID describe a circle.

143. Construct a triangle with base 4·5″ and base angles 45° and 60°, making these angles by construction.

144. Draw a map of a field $ABCD$ from the following measurements: AB 600 m., BC 250 m., CD 390 m., DA 520 m. and AC 650 m. Measure the angles ABC and ADC, and calculate the area of the field in hectares (1 hectare = 10,000 square metres).

145. Construct a square, side 2·9″, and describe a circle to pass through its angular points.

146. Draw a triangle base 3·3″ and base angles (which are to be constructed) 30° and 60°. Find its area.

147. A and B are two outposts, B being 5 miles due South of A. An opposing force is signalled from A to be 20° South of East, and at the same time from B to be 35° North of East. How far is the force from the nearer outpost?

148. Pattern No. 13.

149. The size of a four-sided field *ABCD* is to be determined by the following measurements: *AB* 100 metres, *AC* 90 m., *AD* 80 m., $\angle BAC$ 90°, $\angle BAD$ 120°. Draw a diagram of the field, and reduce it to a triangle of equal area. Find the area of the field in square metres.

150. Make an isosceles triangle with base 5 cm., and equal sides 4 cm. Through each angular point draw, by construction, lines parallel to the opposite sides, and produce them to meet.

151. Draw four parallel straight lines 1″ apart and, cutting these at right angles, five others ·7″ apart.

152. Make a rectangle whose diagonal is 4″ and one of whose sides is 3″. Calculate its area in sq. inches.

153. A man cycles W. 26 miles, N. 9 miles and W. 14 miles. Calculate his distance in a direct line from his starting point. Check by measurement.

154. Pattern No. 14.

155. An upright pole $5\frac{1}{2}$ ft. high stands 4 ft. from a lamp-post, and the shadow of the pole cast by the lamp is $3\frac{1}{2}$ ft. long. Draw a diagram to scale and find the height of the lamp.

156. *ABC* is a triangle with *a* 10·5 cm., *b* 97 mm. and *c* 1·2 dm. Calculate its area in cm². in 3 ways. Take the average of these results.

157. Draw a triangle with sides 3″, 4·1″, 3·7″. With longest side 8·2 cm. draw a similar triangle by copying the angles.

158. Draw a parallelogram with sides 9·5 cm. and 6·9 cm. and angle 115°. Bisect the sides. What is the quadrilateral formed by joining the middle points of the sides of the parallelogram?

159. Pattern No. 15.

160. Draw a plan of a field *ABCD* from the following measurements: *AB* 600 yds., *BC* 500 yds., *CD* 700 yds., $\angle ABC$ 135° and $\angle BCD$ 60°. Measure *AD*. Construct a triangle equal in area to the quadrilateral, and find the area of the field. [Scale 200 yds. to the inch.]

161. Draw a triangle *ABC* having *a* 4·2″, *b* 3·1″, *C* 87°. Construct a parallelogram equal in area to this triangle, and having an angle equal to half the angle *ACB*.

162. To construct a plan of the path followed in ascending Mt. Blanc from Chamounix :—Draw a line CB to represent the total distance on the map 13 kilometres. Along this line take points to represent the following distances from Chamounix, 3·5 Km., 4 Km., 5·5 Km., 8 Km., 10 Km., 12 Km., 13 Km. Through these points draw a series of lines at right angles to CB, and along them mark off lengths to represent the following heights : 1050 m. (Chamounix), 1850 m., 2050 m., 2370 m., 3050 m., 3930 m., 4670 m., 4810 m. (the summit). A curve drawn through the points so obtained will show the steepness of the ascent.

163. ABC is a triangle with a 4″, b 4·7″ and c 5·2″. Construct a similar triangle, with shortest side 8 cm., by copying the angles. Measure the longest side of this triangle.

164. Construct (i) a square, diagonal 3″ ; (ii) a rectangle, diagonal 9 cm. and one side 5 cm. Measure the sides of each figure.

165. ABC is a triangle with C 43°, a 5″ and b 4·5″. At every inch along BC draw parallels to BA. Measure the parts into which CA is divided.

166. Pattern No. 16.

167. What is the locus of a point equidistant from two given points ? A house is to be built in a park $ABCD$ equidistant from its four corners. ($AB = 800$ yds., $BC = 460$ yds., $CD = 720$ yds., $DA = 1060$ yds. and the diagonal $BD = 1000$ yds.) Is it possible to do this ? Find its position if it is equidistant from A, B and C. Measure its distance from D.

168. The sides AB, BC, CD, DE, EA of the figure $ABCDE$ are of lengths 35, 84, 48, 36, 109 mm. respectively. The angles at B and D are right angles. Find the area of the figure.

N.B.—Begin by proving that ACE is a right-angled triangle.

169. Pattern No. 17.

170. Draw a triangle ABC in which $a = 10$ cm., $b = c = 7·5$ cm. Let the bisectors of the angles B and C meet in O. Prove OA bisects the angle BAC. With centre O, and the perpendicular let fall from O on BC as radius, describe a circle to touch the sides of the triangle ABC.

171. Draw a rhombus $ABCD$ with sides 2″ and angle 35°. Draw lines AE, BF, CG, DH, each 2″ long, at right angles to AB. Join EF, FG, GH, HE so that the whole figure represents a cube.

172. Draw a line 4″ long to represent 3 miles. Divide this line into 3 equal parts, and the left-hand division again into 8 equal parts. Mark off this scale in miles and furlongs.

173. Use the scale of the previous question to draw the map of an estate $ABCD$ in which $AB = 2$ m. 6 f., $BC = 6$ f., $AD = 1$ m. 2 f. $\angle BAD = 45°$ and $\angle ABC = 60°$. Measure CD in miles and furlongs; and, by drawing an equivalent triangle, calculate the area of the estate in acres.

174. Draw a right-angled triangle with sides 7 cm. and 9 cm. Measure the hypotenuse, and thence find the square root of 130.

175. Draw a straight line 12·5 cm. long and divide it into 3 equal parts. Hence draw an equilateral triangle with perimeter 12·5 cm.

176. Pattern No. 18.

177. Draw a right-angled triangle with hypotenuse 1·14 dm. and one side 77 mm. Calculate the remaining side approximately, and check by measurement.

178. Draw a circle with radius 5·4 cm. Construct four diameters making angles of 45°, and join their extremities to form a regular octagon. Find the area of one of the triangles into which it is divided, and so find the area of the octagon.

179. On a map 3 miles is represented by 3·4″. Draw a scale to read miles and furlongs. (Show 5 miles.) Mark with two small crosses a distance of 3 m. 5 f.

180. Draw a triangle ABC with a 7·2 cm., b 8·3 cm. and C 72°. On a base 10 cm. make a triangle of the same area having one angle also 72°.

181. Divide a straight line 4·9″ long into three equal parts and measure them in mm.

182. Describe a triangle ABC with c 63 mm., B 65°, C 39°. Bisect two sides at right angles by straight lines meeting at O. Prove that $OA = OB = OC$. Check by drawing a circle, centre O and radius OA.

183. Pattern No. 19.

184. A cyclist riding North along a straight road observes a windmill at an angle 30° W. of N. After two miles the windmill is exactly N.W. How far is the windmill from the nearest point of the road?

185. In a scale 7 ft. is represented by 1". By what length would 40 ft. be represented? Draw a scale to show 40 ft. by which single feet may be measured.

186. Using the scale of the previous question draw the quadrilateral $PQRS$ in which $PQ = 27$ ft., $QR = 26$ ft., $RS = 12$ ft., $SP = 15$ ft. and $PR = 25$ ft. Reduce it to an equivalent triangle and find its area in sq. feet.

187. Pattern No. 21.

188. On a map 300 miles is represented by 3·75". Draw a scale for the map showing 400 miles, and divide it to read distances of ten miles.

189. Using the scale of the previous question draw a map of the State of Colorado, which is a rectangle measuring 380 miles E. and W., and 280 miles N. and S. Find the area of Colorado in sq. miles.

190. The breadth of a river between two points A and B on the banks is 80 feet. Soundings are taken at intervals of 10 ft. along AB, and the depths at those points are found to be 2 ft., 4½ ft., 8 ft., 15½ ft., 18 ft., 15 ft. and 8½ ft. Draw a figure to show the section of the river-bed between A and B: First join the points giving the various depths by straight lines, and then draw a curve through these points freehand.

191. In the triangle ABC $a = 9{\cdot}5$ cm., $b = 11{\cdot}2$ cm., $C = 65°$. Construct a parallelogram equal in area to the triangle and with one angle 73°.

192. What is the locus of a point which is equidistant from two intersecting straight lines?

A triangular park ABC with sides 4200 yds., 3700 yds. and 3100 yds. is bounded by three roads; and in it a house is situated at the same distance from each road. Draw a diagram to show the position of the house, and find how far it is from each road.

193. Two ships C and D are observed at the same time from the two ends A and B of a base line 1000 yards long. If CAB and DAB be 60° and 30°, and CBA and DBA be 50° and 80°, find (by drawing a figure to scale) the distance between the ships.

194. Pattern No. 25.

195. Find the area of a field *ABCDE* from the following measurements: *AD* is 600 yards, and along it are measured *AX* 230 yds., *AZ* 350 yds. and *AY* 470 yds. At *X* and *Y* offsets *XB* 240 yds. and *YC* 160 yds. are measured perpendicular to *AD* and on the same side of it; at *Z* on the other side of *AD* an offset *ZE* 220 yds. is measured perpendicular to *AD*.

196. Construct a square with sides 3·2″ long. Let fall a perpendicular from the point of intersection of the diagonals on to one of the sides of the square. Describe one circle inside the square, and another outside.

197. Pattern No. 24.

198. Draw a triangle *ABC* having *a* 3″, *A* 40° and *B* 105°. Construct a second triangle equal in area to *ABC* on a base 6 cm. and having an angle equal to half the angle *ABC*.

199. Pattern No. 26.

200. On a map lengths are reduced 1 : 3000. Draw a scale of yards for the map, showing 500, and reading to 10 yards.

201. *ABCDE* is a field having the following dimensions: *AB* 250 yds., *BC* 280 yds., *CD* 230 yds., *DE* 180 yds., *EA* 300 yds., *AC* 390 yds. and *AD* 400 yds. On the scale of the previous question, draw a diagram of the field, reduce it to an equivalent triangle and find its area in sq. yds. and in acres.

202. Draw a section of a river-bed from *A* to *B*. The width *AB* is 45 ft., and at intervals of 5 ft. soundings are taken and the depths in feet found to be 0, 1, 3, 7, 5, 10, 16, 18, 13, 0.

203. *ABC* is a triangle in which $a = 7{\cdot}9$ cm., $B = 64°$, $C = 59°$. On a base 5·2 cm. make a parallelogram, of equal area, with one angle 49°.

204. Pattern No. 22.

PATTERNS.

The dotted lines must be drawn as lightly as possible.

1. All radii 2″.

2. Radii of all circles 1·4″.

3. "Step" a distance ½″ along a straight line. Take 8 steps. The radius of the circle is 4 steps. The radii of the semicircles are 1, 2 and 3 steps.

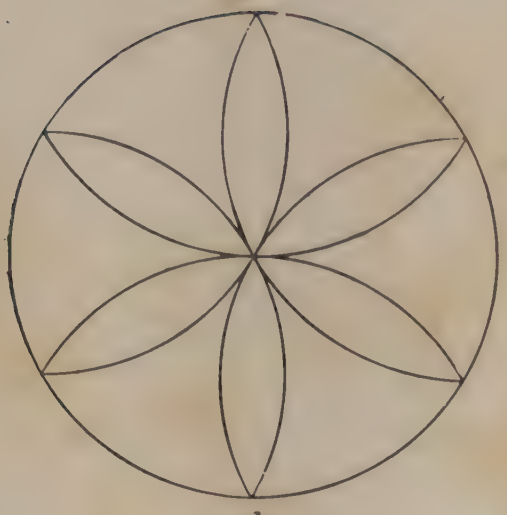
4. The Trefoil. Equilateral triangle, sides 2″. Radii 1″.

5. Equilateral triangle, sides 5 cm. Radii of arcs outside the triangle 1·5 cm. and inside the triangle 3·5 cm.

Variation.

Same triangle, but inside radii 1·5 cm. and outside radii 3·5 cm.

6. Equilateral triangles, sides 4 cm. All radii 2 cm.



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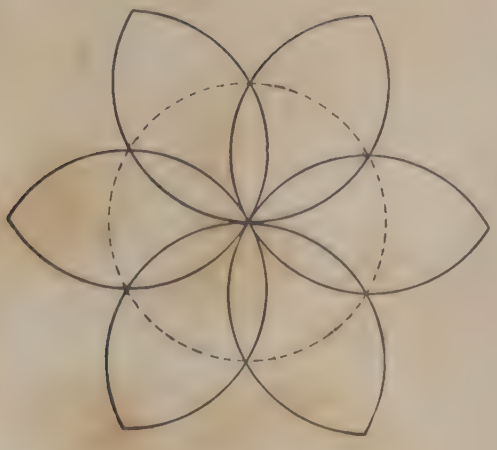

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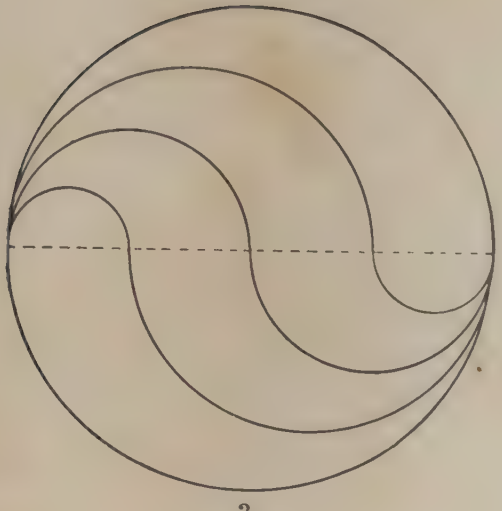

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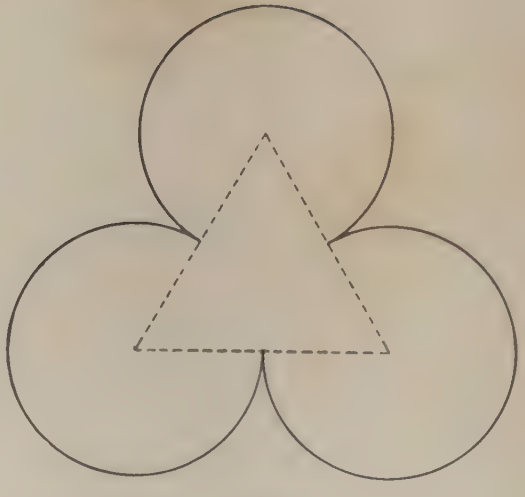

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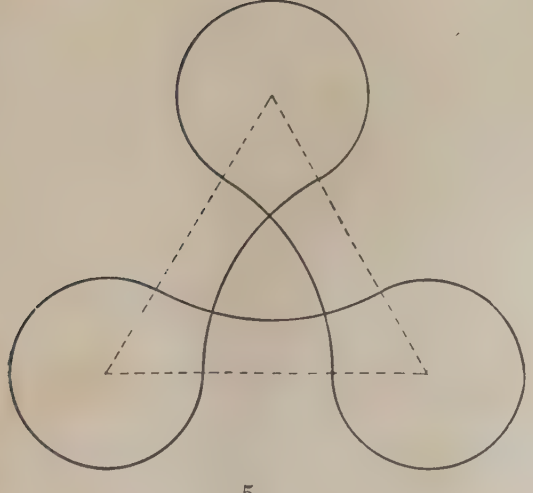

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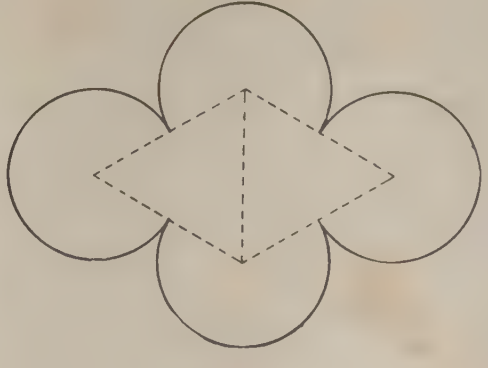


6.

7. Radius of circle 5 cm. Join alternate points of the hexagon.

8. Circle, radius 2″; radii of arcs 1″ (*inside* the hexagon).

Variation.

Circle, radius 1·4″; radii of arcs ·7″, but draw them *outside* the hexagon.

9. Radius of circle 1·4″. Draw the hexagon. Radii of outside arcs ·8″ and inside arcs ·6″.

Variations.

(*a*) Same circle and hexagon. Radii of outside arcs and inside arcs each ·7″, with centres at alternate points.

(*b*) Same circle and hexagon. Radii of outside arcs ·6″ and inside arcs ·8″.

10. Sides of equilateral triangle 6 cm. Bisect the sides. Semicircles (*inside*) radii 1·2 cm. Other radii 1·8 cm.

Variations.

(*a*) Semicircles (*inside*) radii 1·5 cm. Other radii also 1·5 cm.

(*b*) ,, ,, 1·8 cm. Other radii 1·2 cm.

11. Sides of equilateral triangle 2·4″. Bisect the sides. Semicircles (*outside*) radii ·7″. Other radii ·5″.

Variation.

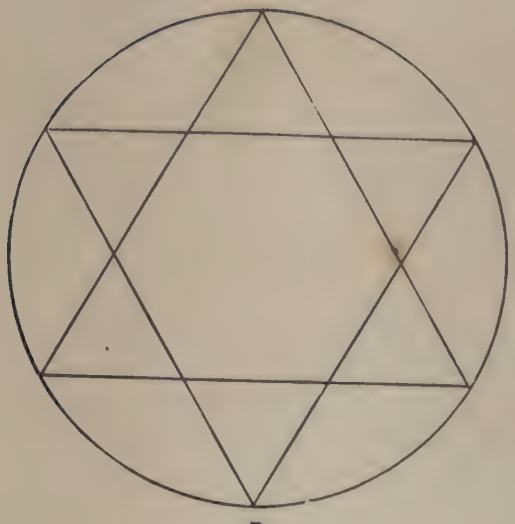
Semicircles (*outside*) radii ·7″. Other radii 1·9″.

12. Sides of equilateral triangle 4 cm. Bisect the sides. Semicircles (*inside*) radii ·8 cm. Other radii 2·8 cm.

Variations.

(*a*) Semicircles (*inside*) radii 1 cm. Other radii 3 cm.

(*b*) ,, ,, 1·3 cm. Other radii 3·3 cm.



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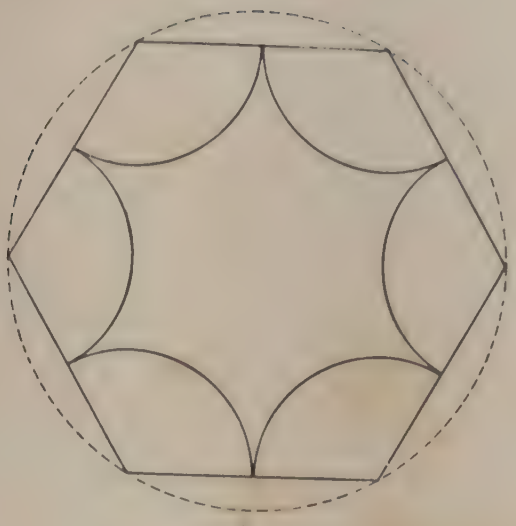

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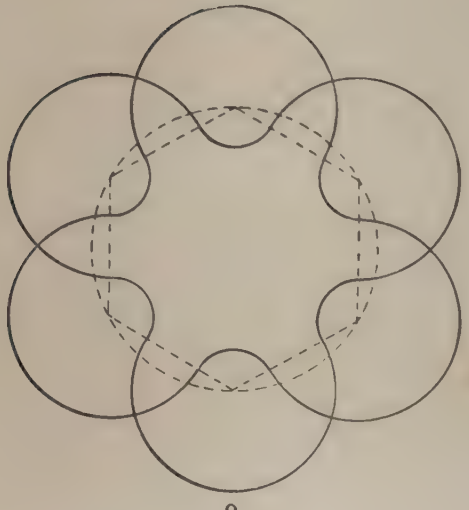

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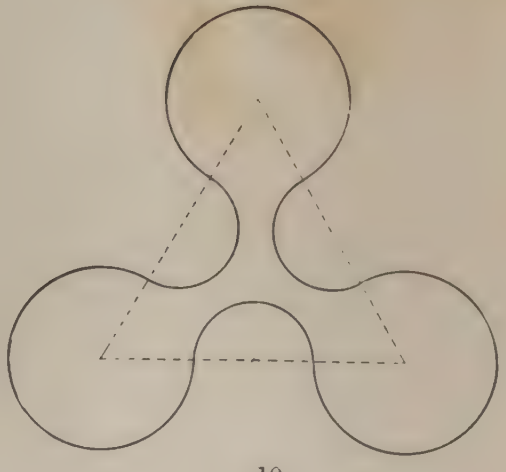

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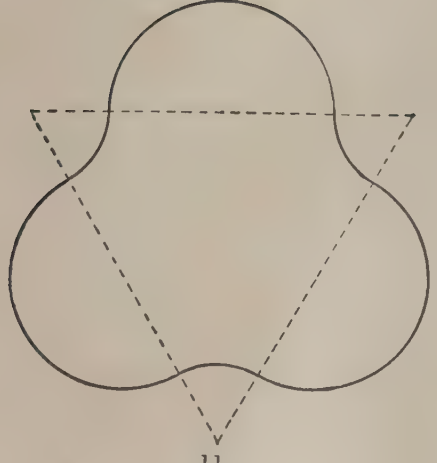

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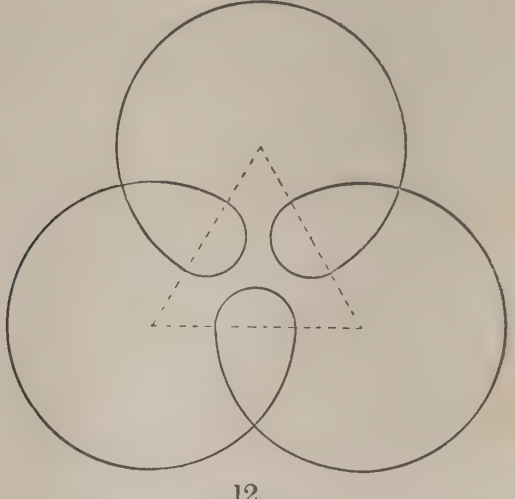


12.

13. Radius of circle 4 cm. Draw the hexagon. Bisect its sides. Semicircles (*inside*) radii. 1 cm. Other radii also 1 cm.

Variations.

(*a*) Semicircles (*inside*) radii ·8 cm. Other radii 1·2 cm.

(*b*) Semicircles (*inwards*) radii 3 cm. Other radii 1 cm.

[The sides of the hexagon should be produced in this case.]

14. Radius of circle 1·2″. Draw the hexagon. Bisect its sides. Semicircles (*inside*) radii ·3″. Other radii ·9″.

Variation.

Semicircles (*inwards*) radii ·8″. Other radii 1·4″. [The sides of the hexagon should be produced in this case.]

15. Circle, radius 2″. Two diameters at right angles. Draw semicircles on the radii.

16. Square, sides 5 cm., radii of arcs *inside* the square 3 cm. and *outside* 2 cm.

Variation.

Radii of arcs *inside* 2 cm. and *outside* 3 cm.

17. Square, sides 5·6 cm. Bisect the sides. Semicircles (*inside*) radii 1·4 cm. Other radii also 1·4 cm.

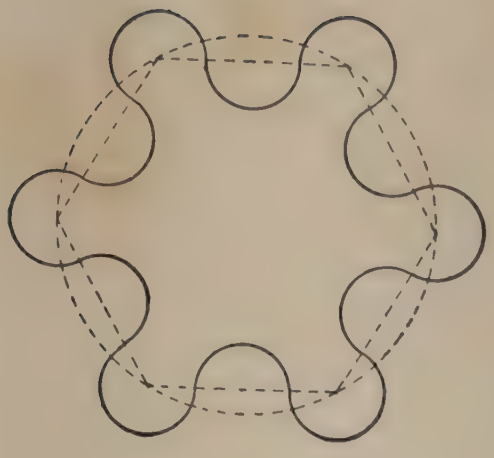
Variations.

(*a*) Semicircles (*inside*) radii 2·3 cm. Other radii ·5 cm.

(*b*) Semicircles (*inwards*) radii 3·8 cm. Other radii 1 cm.

[The sides of the square should be produced in this case.]

18. Square, sides 2″. Bisect the sides. Semicircles (*outside*) radii ·5″ and 1″. Other radii ·5″.



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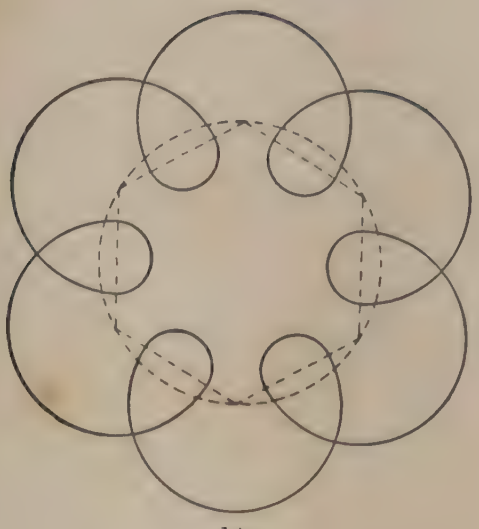

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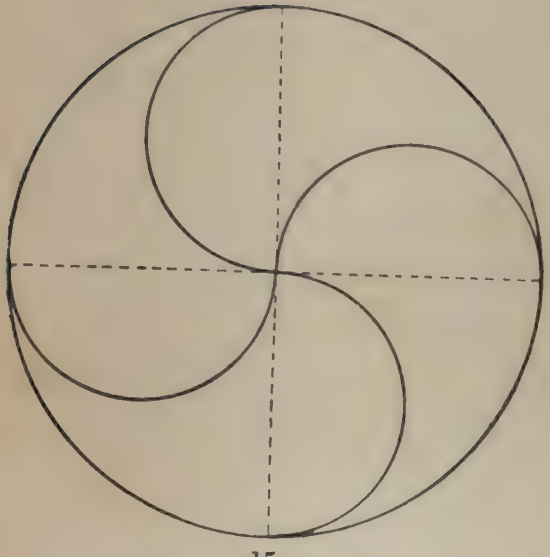

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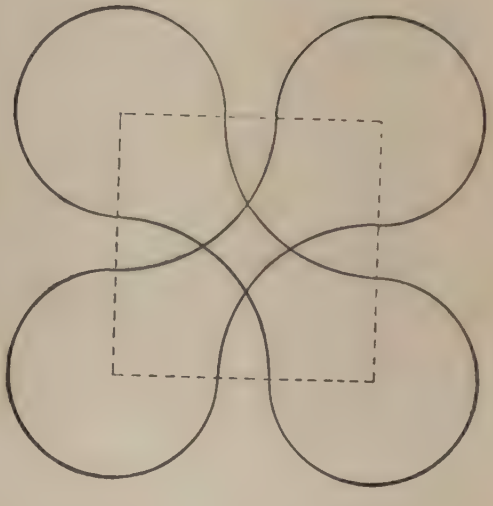

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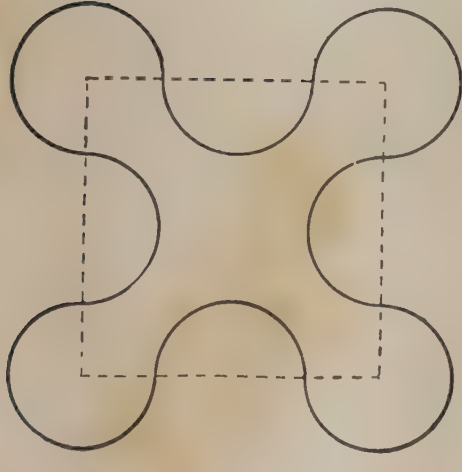

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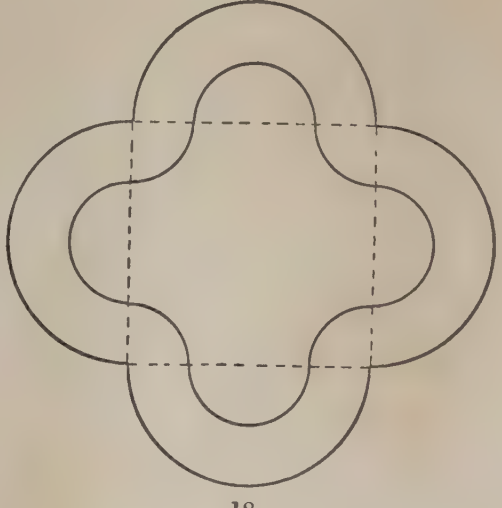


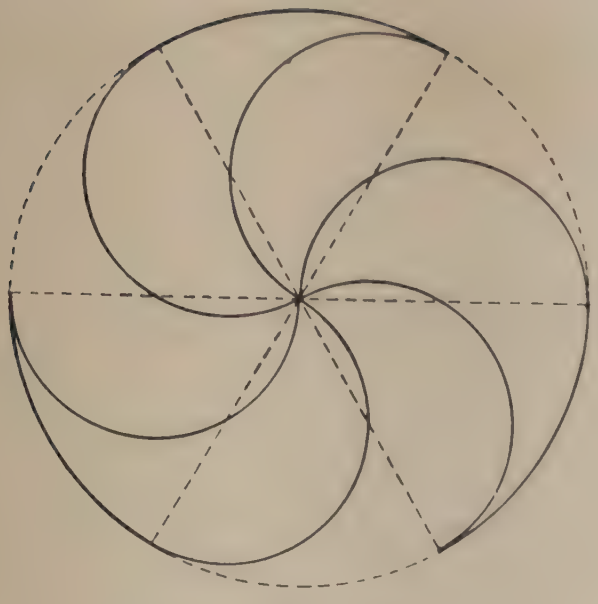
18.

19. Circle, radius 2″. Draw the six radii. Bisect them. Describe semicircles (radii 1″).

20. Radius of dotted circle 1·2″. Draw the hexagon, and the six radii. Produce them. Small radii ·6″. Surround with a large circle.

21. The Maltese Cross. Draw two lines at right angles. Make *OA* 1″ and *OB* 2″, etc. With centre *B* and radius 1·8″ describe arcs, etc.

22. The Honeycomb. A hexagon should be placed in a circle of radius 2 cm. and on each of its sides another hexagon described.



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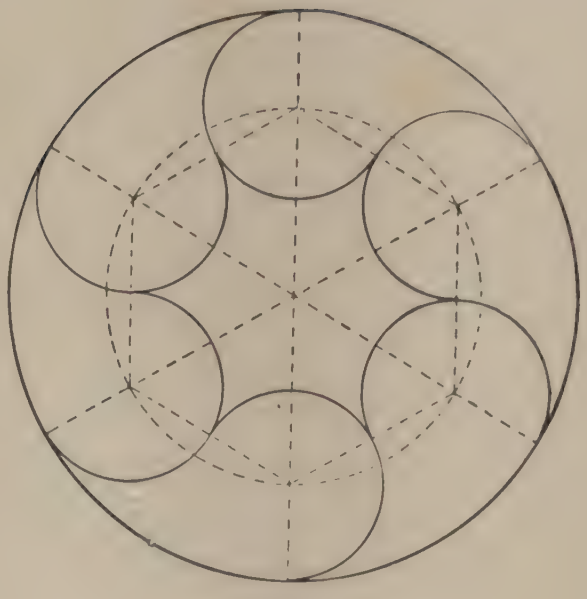

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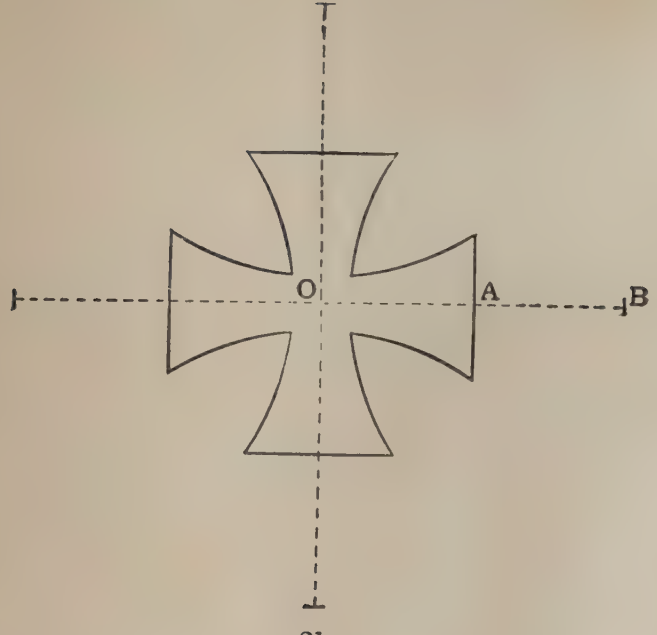

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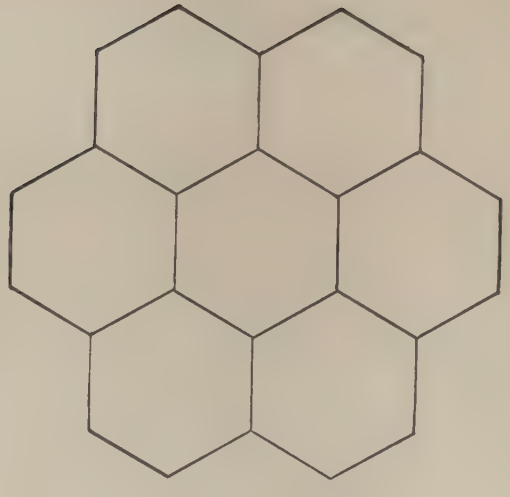


22.

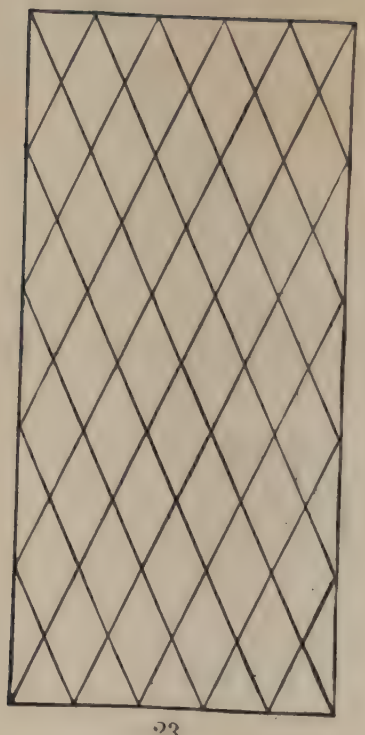
23. Rectangle with sides 13 cm. and 6 cm. Divide a long side (by construction) into 5 equal parts. Through the points of division draw straight lines parallel to the diagonals.

24. Interlacing Pattern. Circle radius 1·6″. Find the angular points of the inscribed hexagon. Use them as centres and take radii 1″ and 1·1″. Make the bands go alternately under and over.

N.B.—Any of the following patterns are suitable for elaboration into interlacing patterns. Nos. 5, 7, 9, 10, 11, 12, 13, 14, 16, 17.

25. The base is 6 cm. The radii of the intersecting arcs are 3 cm. The other radius is 4·5 cm.

26. Cross section of a rail. All the dimensions are in inches.



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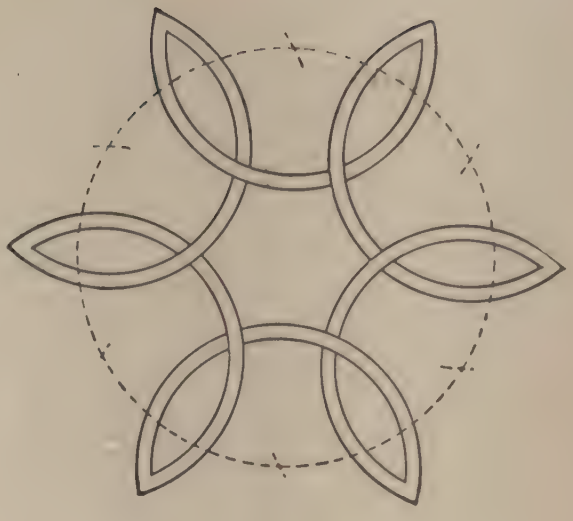

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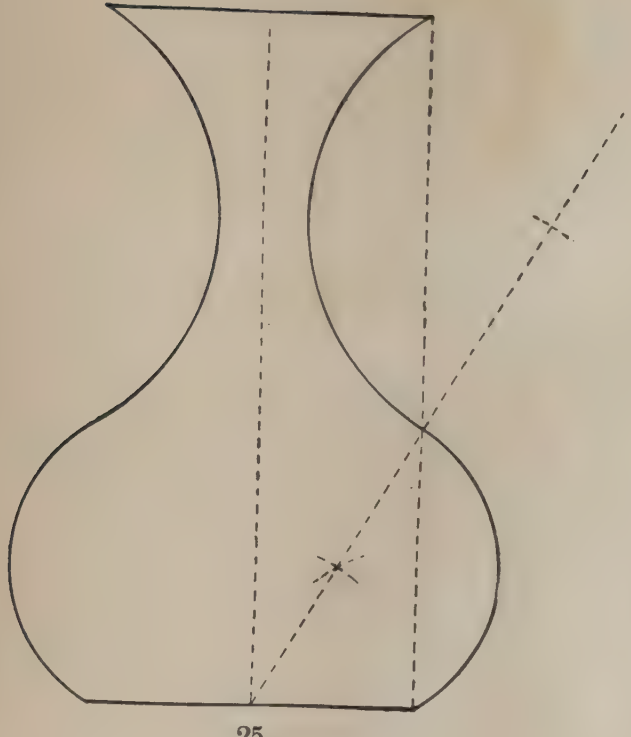

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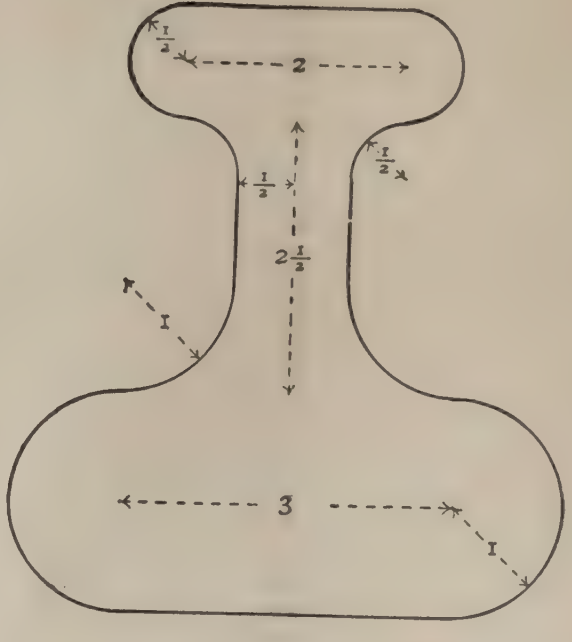

26.

APPENDIX.

ALLCOCK, PART I.	EUCLID, BK. I.	
Prop. 1	Prop. 13	viii. 1-4; xix. 1, 2, 4, 5.
2	14	viii. 12, 13.
3	15	viii. 2, 4.
4	4	ix., especially 12.
5	5	viii. 6, 7; ix. 3, 4; xiv. 2, 3, 5.
6	6	ix. 7, 8, 15; xiv. 9.
7	8	iv.
8	26 (i)	ix., especially 13.
10	18	xiv. 1, 6; xix. 7.
11	19	ix. 10; xiv. 7; xix. 6.
12	20	iv. 8.
13	21	xix. 8.
14	24	ix. 11.
15	25	xiv. 18.
16	27	xx. 14-16; xxi. 1.
17	28 (i)	xxi. 2, 3.
18	28 (ii)	xxi. 4, 12.
19	29	xxi. 13.
21	32	viii. 5, 8, 9; xiv. 10-12, 20; xix. 9-15.
22	26 (ii)	xiv. 10-12.
24		xiv. 13-16.
25	33	xxi. 15, 17.
26	34	xxi. 14; xxiv. 10.
27	35	xxii. 13-16.
28		xxiv.
29	41	xxii. 10-12; 18-22.
31	47	xxv.
32	48	xiv. 29; xxv.
33		xi. 24-28; xvi. 22-30.
34		xii. 10, 12, 15; xviii. 17-19.
35	1 and 22	iv.
36	9	x.; xv.
37	10	v.; ix. 17-21; xi. 23-28; xvi.
38	11	xi.; xvii.
39	12	xii.; xviii.
40	23	xx.
41	31	xxi.
42	46	xvii. 13-16; xxiv.
43		xxvi. 1-8.
44	42	xxvi. 9-14.
45		xxvi. 15-20.
46		xxvii.
The angle in a semicircle is a right angle.		viii. 10; xvi. 18-20. xviii. 10-12; xix. 16. xxiv. 14-22.

ANSWERS.

PART I.

EXERCISE II. Pages 2–5.

8. 14·6 mm. 9. ·52″. 17. 18. } 1″ = 2·54 cm. 19. 20. } 1 cm. = ·394″.

EXERCISE IV. Pages 6–8.

6. 7·62 cm. 11. 12·1 cm. 12. 3·73 cm.

EXERCISE V. Pages 9, 10.

14. 10·2 cm. 15. 2·4″, 1·8″, 1·4″.

EXERCISE IX. Pages 17–19.

1. 3·09″, 48°, 82°. 2. 9·81 cm., $67\frac{1}{2}°$, $47\frac{1}{2}°$. 3. $A = B = 53\frac{1}{2}°$.
4. 114°, 114°. 5. $B = 35\frac{1}{2}°$, $C = 29\frac{1}{2}°$. 6. 3·54″, 4·34″, 60°.
8. $b = c = 4·48″$, 113°. 9. 53°, 7·96 cm., 11·85 cm. 10. 36°, 3·75″.
11. 1·61″, 2·19″, 3·4″. 15. $AB = AC = 3·24″$. 16. Right-angled triangle, 90°.
17. 2·45″, 90°. 18. 7·68 cm. 19. 36°, 64°.
21. 1·9″. 23. 41 ft. 24. 2·98″, 2·13″, 4·06″.
25. 23 m. 26. 248 ft.

EXERCISE X. Pages 19–21.

10. 90°. 12. $BD = CD = 47·4$ mm.
13. $BD = 4$ cm., $CD = 6$ cm. 14. $AF = BF = 2·58″$.

EXERCISE XI. Pages 21–24.

11. 2·68 cm. 12. 3·82″. 13. 2·4″.
16. 2″ and 1·8″. 17. 123°. 18. Rhombus, side 2·5″.
19. Rhombus, side 3·05″. 20. 4 cm. 21. $AF = BF = 2{\cdot}6''$, $AFB = 60°$.
26. $AX = BX = 2{\cdot}9''$. 27. $SA = SB = SC = 5{\cdot}13$ cm.

EXERCISE XII. Pages 24–26.

3. 1·5″. 9. 40°. 11. 3·6 cm. 12. Equal.
13. 125°. 17. 150 ft. 18. Shortest distance = ·413 mile.

PART II.

EXERCISE XIII. Pages 27, 28.

1. 3·33 miles. 3. 4·94 miles. 4. 29·45 miles.
5. 90·8 yards. 7. 3·6 miles. 8. 388 yards.

EXERCISE XIV. Pages 28–30.

1. $A = 83°$. 2. $A = B = 70°$. 3. 60°, 60°, 60°.
4. 11·1 cm. 5. 2·65″, $65\frac{1}{2}°$, $65\frac{1}{2}°$. 6. 34°, 41°.
7. 2·94″, 3·41″. 8. 3·42″. 9. $a = c = 10{\cdot}2$ cm.
11. 2·21″. 12. 11·3 cm. 15. $74° + 106° = 180°$.
18. 71°, 67°. 19. $70\frac{1}{2}°$. 22. 78°, 61°.
23. 10·5 cm., 121°. 24. 4·27″. 26. 115°.
27. 140 mm. 28. 121°, 59°, 121°. 29. 90°.
30. ·57 mile. 31. $BD = 409$ yds., $EC = 220$ yds., $ED = 277$ yds.

EXERCISE XV. Pages 30–32.

12. $BO = CO = 4{\cdot}81$ cm., $BD = CE = 7{\cdot}64$ cm.
24. 60°, 120°, 90°, 270°, 45°, 45°. 26. 1·53″.

EXERCISE XVI. Pages 32–35.

3. } $DE = \frac{1}{2}BC = 1{\cdot}5''$. 7. 9·84 cm. 12. Rhombus, sides 5·5 cm.
4. }
13. 104°. 19. At A and C. 28. 1·9 miles.
30. 187 yards. 31. 2·65″. 33. 7·35 cm.

EXERCISE XVII. Pages 36–38.

3. 3″, ·6″. 4. 6 ft. 6 in. 5. 250 yds. 6. 125 m.
8. 32° or 148°. 9. 2″. 10. 3·96″. 14. 3·7″.
15. 4·24″. 18. 4 cm. 19. 2″. 20. 30°.
21. 11·7 m. 24. 64°. 25. 8·73 cm. 26. 131 mm.
29. Length of hedge, 12·5 m. 30. 34″.

EXERCISE XVIII. Pages 38–40.

4. 2·4″. **5.** 1·55″, 2·21″. **6.** 180 ft. **7.** 45° or 135°.
8. 346 yds. **9.** 37·5 m. **17.** ·99″. **18.** 3·14 cm.

EXERCISE XX. Pages 42, 43.

4. 5·4 cm. **5.** 7 cm. **6.** 4·35″.
7. 74 mm. **13.** 73°, 62°, 11°.

EXERCISE XXI. Page 45.

19. 15 ft. beyond the taller pillar.

EXERCISE XXII. Pages 46–49.

11. 4·2 sq. in. **12.** 30 cm². **14.** 7·87 sq. in.
15. 4·13 sq. in. **16.** 25 cm². **17.** 24 cm².
18. 8·4 cm²., 11·76 cm²., 20·16 cm². **19.** 17·3 cm². **20.** 2·1 sq. in.
21. (i) 3·6 sq. in.; (ii) 21·5 cm².; (iii) 28·8 cm².
22. 24 cm². Area of circle = 78·5 cm².

EXERCISE XXIII. Pages 49–51.

1. (i) 53·6 cm². (ii) 7·32 sq. in. (iii) 26·5 cm².
(iv) 1·875 sq. in. (v) 31·5 cm². (vi) 9·46 sq. in.
2. (i) 6·71 sq. in. (ii) 49·1 cm². (iii) 5·62 sq. in.
(iv) 3000 mm². (v) 4·39 sq. in. (vi) 19·2 cm².
3. 64·95 cm². **4.** 9·16 sq. in.
5. (i) 58·4 cm². (ii) 9·41 sq. in. (iii) 52·8 cm².
(iv) 5·03 sq. in. (v) 42 cm². (vi) 11·8 sq. in.
6. (i) 35 cm². (ii) 4·2 sq. in. (iii) 34·6 cm². (iv) 4·35 sq. in.
7. (i) 8·28 sq. in. (ii) 55·5 cm². (iii) 12·7 sq. in. (iv) 93 cm². (v) 10·04 sq. in.

EXERCISE XXIV. Pages 51, 52.

1. 12 sq. in. **2.** 76 cm²., 7600 mm². **3.** 5625 mm²., 56·25 cm².
4. 5·37″. **5.** 1 sq. in. = 6·45 cm². **6.** 12 sq. in. = 77·4 cm².
7. 1 dm². = 15·5 sq. in. **8.** 120 cm². = 18·6 sq. in. **11.** 104 mm.
13. 6·93 sq. in. **16.** 72 cm². **17.** 50·2 mm.
19. $AD = BC = 4·5$″. **20.** 6 cm., 8 cm., 6 cm.; 48 cm². **21.** 4·8″, 4·8″.

EXERCISE XXV. Pages 53, 54.

3. 5 sq. in. **4.** 50 cm². **5.** 9·22 cm. **6.** 2·1″.
7. 7·14 cm. **8.** 40 ft. **9.** 144 yds. **10.** 35·3 ft.
12. 3·64 sq. in., 1·73 sq. in., 1·91 sq. in. **13.** 35·45 cm²., 5·24 cm²., 30·21 cm².

EXERCISE XXVI. Pages 54, 55.

1.	70·6 cm².	3.	5·96 sq. in.	4.	3680 mm².	5.	8·97 sq. in.		
6.	56·5 cm².	7.	70500 sq. yds.	8.	87200 sq. yds.	9.	4·43 sq. in.		
10.	35·5 cm².	11.	9·51 sq. in.	12.	29 cm².	13.	32·9 cm².		
14.	3·625 sq. in.	15.	6·8 sq. in.	16.	34·8 cm².	17.	29·7 cm².		
18.	3·59 in.	19.	34·9 cm².	20.	7·64 sq. in.				

EXERCISE XXVII. Pages 56, 57.

1.	42·3 mm.	2.	33·9 mm.	8.	47·75 mm.
11.	2·6″, 1·3″.	12.	$DE = 2'' = \frac{2}{3}$ of BC.	14.	2·47″.

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